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# The power potential of tidal currents in channels

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Interest in sources of renewable energy has led to increasing attention being paid to the potential of strong tidal currents. There is a limit to the available power, however, as too many turbines will merely block the flow, reducing the power generated. The maximum average power available from a tidal stream along a channel, such as that between an island and the mainland, is estimated and found to be typically considerably less than the average kinetic energy flux in the undisturbed state through the most constricted cross-section of the channel. A general formula gives the maximum average power as between 20 and 24% of the peak tidal pressure head, from one end of the channel to the other, times the peak of the undisturbed mass flux through the channel. This maximum average power is independent of the location of the turbine ‘fences’ along the channel. The results may also be used to evaluate the power potential of steady ocean currents.

**Keywords:** tidal power; tidal currents; renewable energy

## 1. Introduction

Exploiting ocean tides for energy production is usually discussed in terms of closing off a bay and generating electricity through the release of water trapped at high tide (Prandle 1984; Bernshtein *et al.* 1997). There are also proposals to exploit tidal streams (Gorlov 1995; UK Parliament Select Committee on Science and Technology 2001; Triton Consultants Ltd 2002; Pearson 2004; Peplow 2004; Stone 2004), typically in channels with strong currents, in much the same way as wind energy can be tapped.

The tidal stream may be in the entrance to a bay, or in a channel between, say, an offshore island and the mainland. In both situations it is important to estimate the maximum average power, i.e. the maximum average rate at which energy can be extracted, and it is sometimes assumed that this is given by the average flux of kinetic energy in the undisturbed state through the most constricted cross-section of a channel where the currents are strongest. There is no rigorous basis for this assumption, however, and no formulae exist for what the maximum extractable power might be. What is clear is that little power is generated if only a few turbines are placed in the flow, but that adding too many

turbines chokes the flow and again limits the power. It should be possible to determine the optimum number (and placement) of turbines and the associated maximum average power production.

Garrett & Cummins (2004) examined the use of current turbines in the entrance to a bay, and showed that, over a tidal cycle, the average power produced need not be much less than in a conventional scheme with a dam. The sea level difference from outside to inside the bay, required to drive the turbines and produce power, can come from a difference in the time of high and low tides outside and inside the bay, not just from a difference in the tidal range. Hence, reasonable energy production is compatible with maintaining a tidal range in the bay that is not much reduced from its undisturbed value, thus maintaining the flushing that might be desirable for waste dispersal and aquaculture. Garrett & Cummins (2004) also concluded that there is no simple relationship in this case between the maximum average power and the average kinetic energy flux in the undisturbed state.

In the case of a bay, the sea level outside may be prescribed reasonably independently of the currents in the entrance, but clearly the sea level inside, and hence the pressure head driving the currents in the entrance, is dependent on the currents. In the present paper we consider a different scenario, as for a constricted channel connecting two large bodies of water in which the tides at both ends are assumed to be unaffected by the currents through the channel. Once again, the problem is to determine the maximum average power that can be extracted.

## 2. The model

We consider flow through a channel of variable cross-section (figure 1). The current speed  $u(x, t)$  is assumed to be a function of time  $t$  as well as the position  $x$  along the channel, but independent of the cross-channel position.

The dynamical equation governing the flow is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} = -F, \quad (2.1)$$

where the slope of the surface elevation  $\zeta$  provides the pressure gradient to drive the flow and  $F(x, t)$  represents an opposing force associated with natural friction and possibly the presence of turbines. Taking the frictional force associated with the turbines to be independent of the cross-channel position requires that the turbines be deployed in a uniform ‘fence’ across the flow, with all the water passing through the turbines. This is actually a more efficient scheme than using isolated turbines, as that would lead to cross-channel current gradients downstream and loss of energy as these gradients are smoothed by lateral mixing (Garrett & Cummins 2004). There could, of course, be several fences along the channel, so that the turbine-related part of  $F$  is an arbitrary function of  $x$ .

If the channel is short compared with the wavelength of the tide (generally hundreds of kilometres, even in shallow water), volume conservation implies that the flux  $Au$  along the channel is independent of  $x$  and may be written as  $Q(t)$ . (We neglect small changes in  $A$  associated with the rise and fall of the tide.)

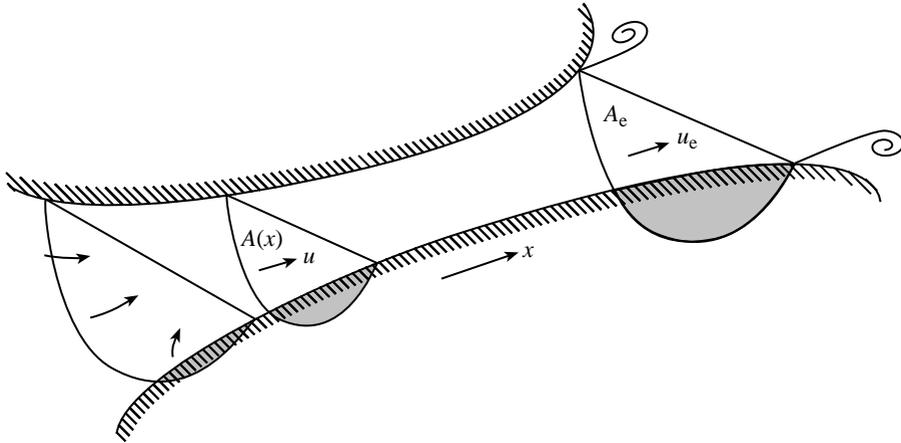


Figure 1. A channel connects two basins with different tidal elevations. At any instant the current through the channel has speed  $u$ , a function of the local cross-sectional area  $A(x)$ , where  $x$  is the along-strait coordinate. At the channel entrance the current enters from all directions, but at the exit it may separate as a jet with speed  $u_e$ , surrounded by comparatively stagnant water with elevation the same as that in the downstream basin.

Using this in equation (2.1) and integrating along the channel implies

$$c \frac{dQ}{dt} - g\zeta_0 = - \int_0^L F dx - \frac{1}{2} u_e |u_e|, \quad (2.2)$$

where  $c = \int_0^L A^{-1} dx$  and  $\zeta_0(t)$  is the sea level difference between the two basins, assuming that this difference is unaffected by the flow through the channel and hence unaffected by changes in  $F$  as turbines are deployed. We note that the geometrical factor  $c$  is insensitive to the locations of the ends of the channel at  $x=0, L$  if  $A$  is large there. This is the case at the channel entrance where flow is drawn in smoothly from a region with large  $A$ , weak currents and prescribed tidal elevation. We allow for flow separation at the channel exit, however, where the flow is likely to emerge as a jet once the channel widens significantly, but with the sea level continuous across the edges of the jet and matching the prescribed downstream elevation. Thus, there is a small sensitivity of  $c$  to the location of the point of separation. More importantly, however, if  $u_e(t)$  denotes the current speed at the exit, there is an associated pressure head loss  $(1/2)\rho u_e^2$ , where  $\rho$  is the density of the water. (The pressure head loss is just  $\rho g$  times the change in elevation. We shall use the term ‘head loss’ to refer to either this change in pressure or the reduction in the sea level.)

There are clearly different basic states, and hence different problems to be considered for tidal power extraction, depending on whether the imposed head difference is balanced in the basic state by the acceleration term on the left-hand side of equation (2.2), or the friction and flow separation terms on the right-hand side. We consider them in sequence and look for general results.

#### (a) *The simplest case*

The natural frictional term and the head loss associated with separation at the exit are likely to be significant, or even dominant, unless the channel is long

and deep. Nonetheless, we start by assuming that these influences are small, so that the natural regime has a balance between the sea level difference and acceleration. We take  $\zeta_0 = a \cos \omega t$ , a sinusoidal tide with amplitude  $a$  and frequency  $\omega$ . This forcing can, of course, be the difference of sinusoidal tides at each end of the channel rather than representing forcing from just one end. The associated volume flux from equation (2.2) is  $Q = Q_0 \sin \omega t$  where  $Q_0 = ga(\omega c)^{-1}$ . The power generated at the turbines is the integral along the channel of the product of water density  $\rho$ , current  $u$ , cross-section  $A$  and the local frictional force  $F$  representing the turbines. Thus, the average power extracted from the flow by the turbines is

$$P = \overline{\int_0^L \rho F Q \, dx} = \rho Q \overline{\int_0^L F \, dx}, \quad (2.3)$$

with the overbar indicating the average over a tidal cycle. This would need to be multiplied by a turbine efficiency factor to give the average electrical power produced. We note that  $P$  is the average power extracted over the whole channel; the factor  $Q$  in equation (2.3) implies integrating over the cross-section and the integral over  $x$  allows for the distribution of the turbine fences along the channel.

We first assume that the drag associated with the turbines is linearly proportional to the current  $u = Q/A$  at any cross-section. Then we may write  $\int_0^L F \, dx = \lambda Q$  and  $P = \lambda \rho \overline{Q^2}$ , where  $\lambda$  is related to the number of turbines and their location along the channel. The governing equation

$$c \frac{dQ}{dt} - ga \cos \omega t = -\lambda Q \quad (2.4)$$

is easily solved to give

$$Q = \text{Re} \left[ \left( \frac{ga}{\lambda - ic\omega} \right) e^{-i\omega t} \right], \quad P = \frac{(1/2)\rho\lambda g^2 a^2}{\lambda^2 + c^2\omega^2}. \quad (2.5)$$

As expected,  $P$  at first increases with  $\lambda$  (more power is generated as more turbines are added), but then decreases as too many turbines choke the flow. The maximum average power, obtained when  $\lambda = c\omega$ , is

$$P_0 = \frac{1}{4} \rho g^2 a^2 (c\omega)^{-1} = \frac{1}{4} \rho c \omega Q_0^2 = \frac{1}{4} \rho g a Q_0. \quad (2.6)$$

The peak flow through the channel is then reduced to  $2^{-1/2}$ , or 71%, of its original value. The head loss associated with the turbines is also 71% of the original head difference along the channel. The maximum average power is independent of the location of the turbine fences along the channel (but with the fewest turbines clearly being needed if they are deployed at the smallest cross-section where the currents are strongest). This result does not depend on the linear representation of the turbine drag.

### (b) Different drag laws

A more realistic representation of the turbine drag would be quadratic in the current speed. We have considered an arbitrary exponent, with drag  $\lambda|Q|^{n-1}Q$

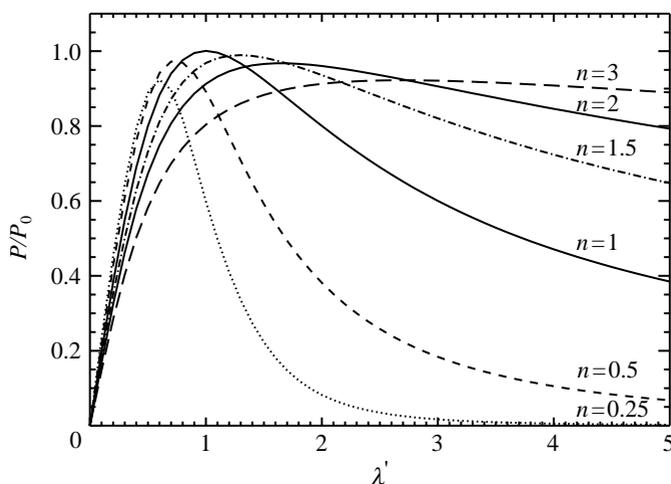


Figure 2. The scaled maximum power as a function of a parameter  $\lambda'$ , representing the frictional drag associated with the turbines, for various values of  $n$  where the turbine drag is assumed proportional to the  $n$ th power of the current speed.

(as for a turbine drag proportional to the current speed to the  $n$ th power), and then non-dimensionalized with  $t = \omega^{-1}t'$ ,  $Q = Q_0Q'$ ,  $\lambda = \lambda'c\omega Q_0^{-(n-1)}$  so that the governing equation is

$$\frac{dQ'}{dt'} - \cos t' = -\lambda'|Q'|^{n-1}Q'. \quad (2.7)$$

We have solved this numerically and evaluated  $P/P_0 = 4\lambda' \overline{|Q'|^{n-1}Q'^2}$  as a function of  $\lambda'$  for different values of  $n$  (figure 2).

The maximum power for  $n=2$  (quadratic drag) is 0.97 times the value  $P_0$  derived for  $n=1$ . Moreover, it does not seem that any other drag law could lead to more power than for  $n=1$ .

We thus take  $P_0$  as a reasonable estimate of the maximum power available in the case of negligible background friction and exit separation effect. We may evaluate it in terms of conditions at the constriction with smallest cross-sectional area  $A_{\min}$ , where the amplitude of the undisturbed tidal current is  $u_{\max}$ . To do this we write  $c = \int_0^L A^{-1} dx = L_{\text{eff}}/A_{\min}$ , where this effective length  $L_{\text{eff}}$  is likely to be considerably less than the total channel length  $L$ . Then

$$P_0 = \frac{1}{4}\rho\omega L_{\text{eff}}A_{\min}^2u_{\max}^2. \quad (2.8)$$

This may be compared with the reference value

$$P_{\text{flux}} = \frac{1}{2}\rho A_{\min} \overline{|u_{\max} \cos \omega t|^3} = \frac{2}{3\pi}\rho A_{\min} u_{\max}^3, \quad (2.9)$$

which gives the average undisturbed kinetic energy flux through the smallest cross-section of the channel and is sometimes used as an estimate of the available

power. The ratio is

$$\frac{P_0}{P_{\text{flux}}} = \frac{3\pi}{8} \frac{\omega L_{\text{eff}}}{u_{\text{max}}}, \quad (2.10)$$

and can either be less than 1 (short channel, strong currents) or greater than 1 (vice versa). For example, if  $\omega = 1.4 \times 10^{-4} \text{ s}^{-1}$ , as for the semidiurnal tide,  $L_{\text{eff}} = 5 \text{ km}$  and  $u_{\text{max}} = 3 \text{ m s}^{-1}$ , then  $P_0/P_{\text{flux}} = 0.3$ , so  $P_{\text{flux}}$  would overestimate the resource. With these values and  $A_{\text{min}} = 5 \times 10^5 \text{ m}^2$  (as for a width of 5 km and a depth of 100 m), then  $P_0 = 800 \text{ MW}$ .

### (c) Including background friction

The calculations so far have neglected background friction and the separation effect. These can be lumped together in a single term if, as is realistic, the friction is quadratic in the current  $u$  and represented in water of depth  $h$  by  $C_d u^2/h$  where  $C_d$  is the drag coefficient, which may be a function of  $x$ . The momentum equation is then

$$c \frac{dQ}{dt} - g\zeta_0 = - \int_0^L F_{\text{turb}} dx - \alpha |Q|Q, \quad (2.11)$$

where

$$\alpha = \int_0^L C_d (hA^2)^{-1} dx + \frac{1}{2} A_e^{-2}, \quad (2.12)$$

and  $A_e$  is the cross-sectional area at the exit. There is a balance at any instant between the acceleration, the surface slope, the drag  $\int_0^L F_{\text{turb}} dx$  associated with the turbines, and the internal friction and separation effect.

If we non-dimensionalize as before and assume that the turbine drag is also quadratic in the local current, equation (2.11) becomes

$$\frac{dQ'}{dt'} - \cos t' = -(\lambda_0 + \lambda_1) |Q'|Q', \quad (2.13)$$

where  $\lambda_0 = g\alpha a(\omega c)^{-2}$  represents the background friction and separation effect and  $\lambda_1$  corresponds to the turbine drag. Here,  $\alpha$  and hence  $\lambda_0$  may be functions of time if  $A_e$  varies with time (particularly between ebb and flood), but we ignore this for the moment.

Note that equation (2.13) is just equation (2.7), which we have already solved, with  $n=2$  and  $\lambda' = \lambda_0 + \lambda_1$ . The power produced by the turbines is just the fraction  $\lambda_1/\lambda' = (\lambda' - \lambda_0)/\lambda'$  times that shown in figure 2 for  $n=2$ , starting at  $\lambda' = \lambda_0$ . The multiplier effectively pulls down the first part of the curve in figure 2, so that its maximum is now a decreasing function of  $\lambda_0$ , as shown in figure 3. This maximum can be used to provide estimates of the average power available in any situation. For large values of  $\lambda_0$ ,  $dQ'/dt'$  can be neglected in equation (2.13), and  $|Q'| = \lambda'^{-1/2} |\cos t'|^{1/2}$ . The average power  $P$  produced by the turbines is then given by  $P/P_0 = 4(\lambda' - \lambda_0)\lambda'^{-3/2} \overline{|\cos t'|^{3/2}}$ , with a maximum of  $0.86\lambda_0^{-1/2}$ , as shown in figure 3. This asymptotic estimate is only 20% too large even for  $\lambda_0 = 2$ .

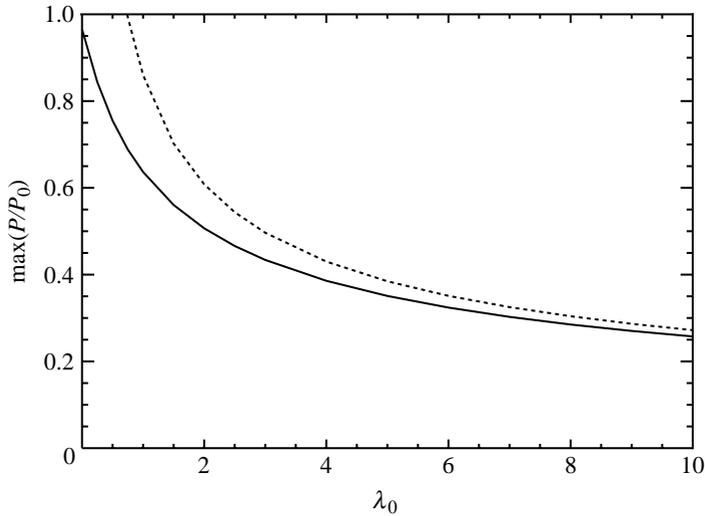


Figure 3. The scaled maximum power as a function of the dimensionless parameter  $\lambda_0$  representing friction and flow separation in the basic state. For large values of  $\lambda_0$ , this maximum power tends to  $0.86\lambda_0^{-1/2}$ , shown by the dashed curve.

(d) *The quasi-steady limit*

This large  $\lambda_0$  limit, in which the acceleration is unimportant, is best appreciated in the dimensional version of the problem. In the absence of turbines, the volume flux  $Q_1$  has magnitude  $|Q_1| = (g|\zeta_0|/\alpha)^{1/2}$  and the same sign as  $\zeta_0$ . With turbines, the power produced at any instant is, using equation (2.11)

$$\rho Q \int_0^L F_{\text{turb}} dx = \rho Q (g\zeta_0 - \alpha|Q|Q). \quad (2.14)$$

This has a maximum as  $Q$  is varied of  $(2/3^{3/2})\rho g Q_1 \zeta_0 = 0.38\rho g Q_1 \zeta_0$ . The flow at any instant is  $3^{-1/2} = 0.58$  of what it was in the absence of the turbines, and two-thirds of the head loss along the channel is now associated with their operation.

The fraction of two-thirds is close to the 71% derived earlier in the linear case with no background friction or separation. It thus seems that the effectiveness of tidal power development in a channel can partly be assessed by considering whether 70% or so of the natural head (typically rather small) can be used efficiently.

For  $\zeta_0 = a \cos \omega t$ ,  $|Q_1|$  may be written as  $Q_{\text{max}}|\cos \omega t|^{1/2}$ . After averaging over a tidal cycle, with  $\overline{|\cos \theta|^{3/2}} = 0.56$ , equation (2.14) leads to an average power of

$$P_1 = 0.21\rho g a Q_{\text{max}}. \quad (2.15)$$

This may be compared with the last formula in equation (2.6), which, after scaling by 0.97 to correspond to quadratic turbine drag, would give a maximum power of  $0.24\rho g a Q_0$ . This shows that, if the maximum power is compared with the peak pressure head along the channel times the peak volume flux in the undisturbed state, the multiplier only changes from 0.24 to 0.21 as we progress from a natural regime with no background friction to one dominated by it.

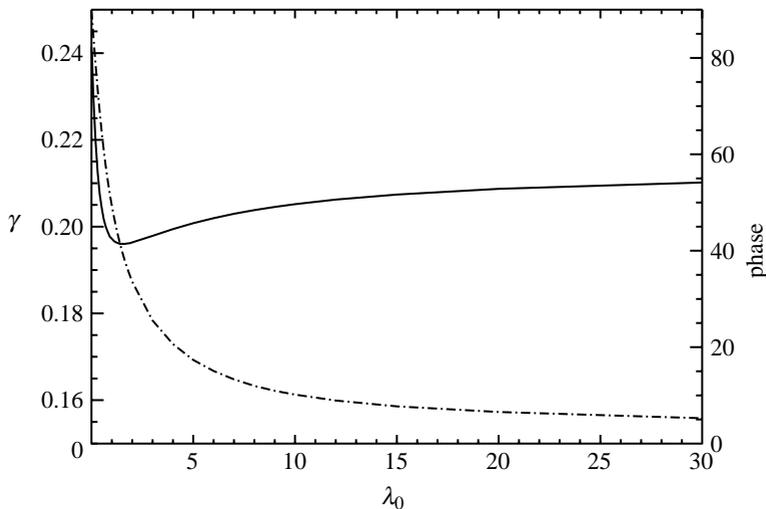


Figure 4. Solid line, left axis: the multiplier  $\gamma$  giving the maximum average power  $\gamma\rho g a Q_{\max}$  as a function of the parameter  $\lambda_0$  representing the effects of friction and separation in the undisturbed state. Dashed line, right axis: the phase lag (in degrees), in the undisturbed state, of the peak volume flux behind the maximum forcing.

The transition is not monotonic, however, as  $\lambda_0$  increases from 0 to  $\infty$ . If we write the maximum average power as  $\gamma\rho g a Q_{\max}$ , we see (figure 4) that  $\gamma$  dips to a minimum of 0.196 for  $\lambda_0=1.4$ . Nonetheless, we conclude that, if data are available on the head and flux in the natural state, the maximum average power available may be estimated with an accuracy of 10% using a multiplier  $\gamma$  of 0.22, without any need to understand the basic dynamical balance. Within the context of our simple model, however, the value of  $\lambda_0$ , and hence a more precise value of  $\gamma$  from figure 4, may be obtained by evaluating the phase lag, in the undisturbed state, of the peak volume flux behind the maximum of the forcing. As expected, and shown in figure 4, this decreases from  $90^\circ$  when  $\lambda_0=0$ , with a balance between the pressure gradient and acceleration, to zero as  $\lambda_0 \rightarrow \infty$ , with a balance between the pressure gradient and friction. The monotonic nature of the decrease means that the phase lag implies a unique value of  $\lambda_0$  and hence of  $\gamma$ .

If the friction within the channel is much less important than the separation effect, equation (2.14) may be written as

$$P_1 = 0.38 \left( \frac{1}{2} \rho A_e \overline{|u_{e0}|^3} \right), \quad (2.16)$$

where  $u_{e0} = (2g\zeta_0)^{1/2}$  is the exit speed at any instant in the natural regime. This formula is similar to equation (2.9), but we note the reduction factor of 0.38 and, particularly, the fact that it must be evaluated at the exit, probably giving a considerably lower value than would be obtained at the most constricted part of the channel. The turbines could still be deployed there, however.

As remarked earlier, the separation point, and hence  $A_e$ , may vary with time, particularly between flood and ebb tide. This is easily allowed for in equation (2.16) by including  $A_e$  in the averaging.

*(e) Non-sinusoidal forcing*

Our analysis so far has been for sinusoidal forcing. For a more complicated tidal regime we may still use equation (2.14) if the balance in the basic state is quasi-steady with maximum instantaneous power proportional to  $|\zeta_0|^{3/2}$ . For example, if we had two tidal constituents, with  $\zeta_0 = a \cos \omega t + a_1 \cos \omega_1 t$  replacing  $a \cos \omega t$ , then the available power is increased from what it would be with  $a_1 = 0$  by a factor which can be shown to be  $1 + (9/16)r^2$  for small values of  $r = a_1/a$ . Actually, this expression is remarkably accurate even for  $r = 1$ , giving a multiplier of 1.56 rather than the true value of 1.57. On the other hand, if the basic dynamical balance is between the pressure head and acceleration, the factor is  $1 + r^2$  for any value of  $r$ . With many constituents of amplitudes  $a_1, a_2, \dots$  and  $r_1 = a_1/a, r_2 = a_2/a, \dots$  the multiplying factor becomes  $1 + (9/16)(r_1^2 + r_2^2 + \dots)$  if the basic balance is frictional, at least for small  $r_1, r_2, \dots$ , and  $1 + (r_1^2 + r_2^2 + \dots)$  if the basic state is frictionless. The conclusion is that the effect of several constituents can be included, but doing so correctly depends on the basic dynamical balance.

### 3. Discussion

The results of this paper can give good preliminary estimates of the power potential of a number of suggested sites around the world, though further consideration for particular sites will require more detailed analysis. One general result is that, to within 10%, the maximum average power available is approximately  $0.22\rho g a Q_{\max}$  for a sinusoidal tidal head  $a \cos \omega t$  and with  $Q_{\max}$  denoting the peak volume flux in the undisturbed state. This upper bound on the available power assumes that turbines are deployed in uniform fences, with all the water passing through the turbines at each fence, but ignores losses associated with turbine operation.

Another factor we have ignored is the back effect on the forcing of changes in the channel flow. The effect is likely to be small if the basins are large and deep, but there may be a positive feedback that will increase the head as turbines are introduced. This will slightly increase the available power.

We have also assumed that the current is independent of the cross-sectional position. If there is shear vertically and across the channel, we may deal with the average speed in the acceleration term, but there will be minor changes in other parts of the theory. These changes will need to be considered in, for example, evaluating the power potential of sheared unidirectional ocean currents such as the Florida Current, but the present theory should provide guidance.

Further analysis is also required for situations where the tidal current is not constricted, as in a channel, but flowing along an open coast. Here, too, there should be optimum designs for a tidal fence, or fences, in part of the flow.

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