The modelling of tidal turbine farms using multi-scale, unstructured mesh models

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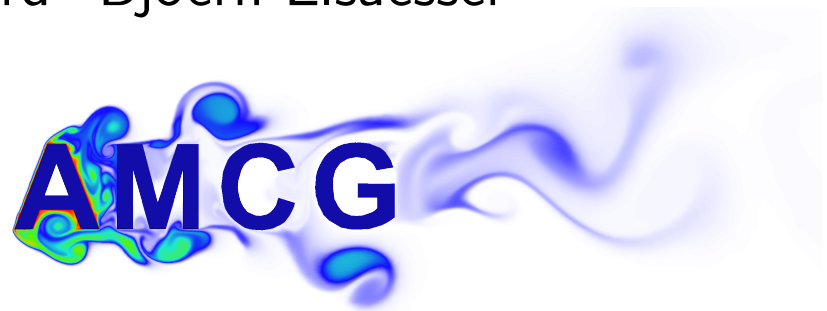

EPSRC
Engineering and Physical Sciences Research Council

SuperGen/UKCMER project:
Large Scale Interactive Coupled Modelling of Environmental Impacts of Marine Renewable Energy Farms (LINC)

Outline

- Benchmarking of Fluidity and MIKE 21 in a simplified tidal basin geometry.
- Implementation of turbines in coarse resolution tidal models.
- Correction to drag coefficient to avoid incorrect mesh dependence

http://amcg.ese.ic.ac.uk/fluidity

Open Source Finite Element Modelling framework used for a variety of Earth science and engineering applcations.

Fluidity solves 3D non-hydrostatic Navier Stokes equations, but also depth-averaged 2D shallow water equations. It implements a host of advanced numerical techniques, e.g. mesh adaptivity which dynamically adapts the mesh to focus resolution where needed.

In development: optimisation framework for farm layouts

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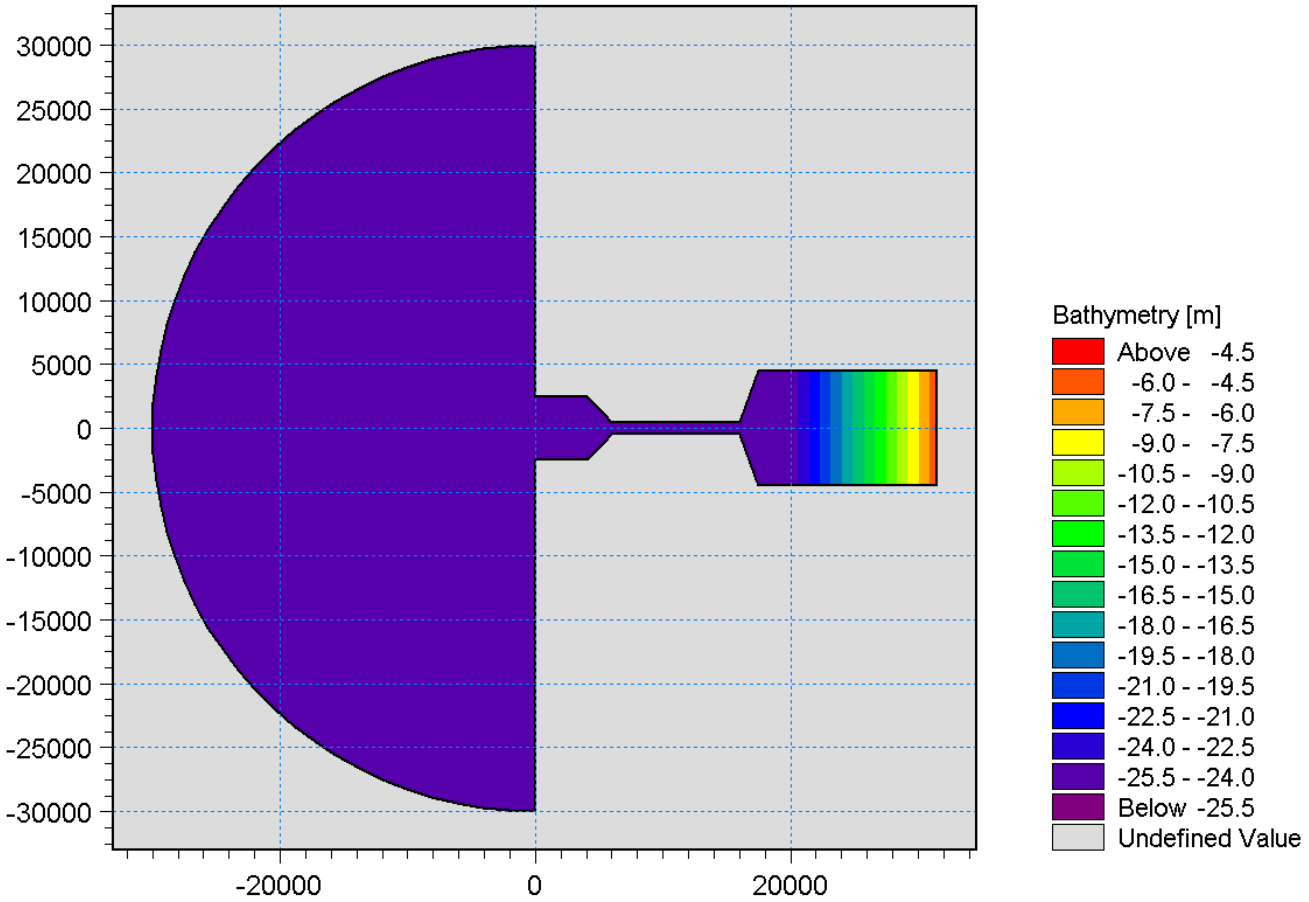
In development: optimisation framework for farm layouts

Widely used in marine engineering.

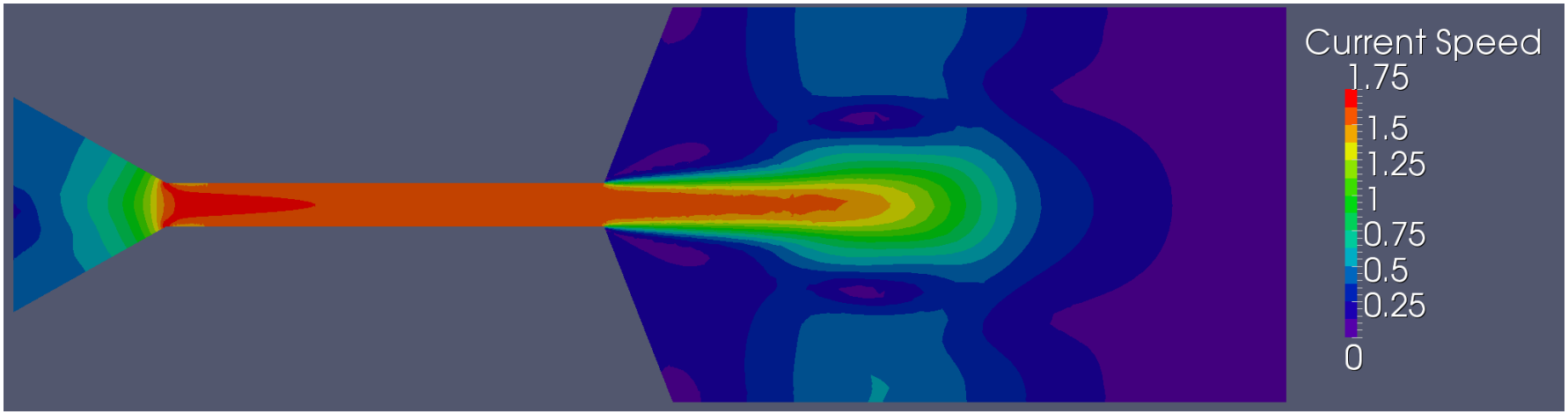
- 3D and 2D capability (MIKE 3, MIKE 21)
- Sediment transport
- Ecological modelling (ECO Lab)
- Wetting and drying
- Structures (including turbines!)

Tidal basin benchmark

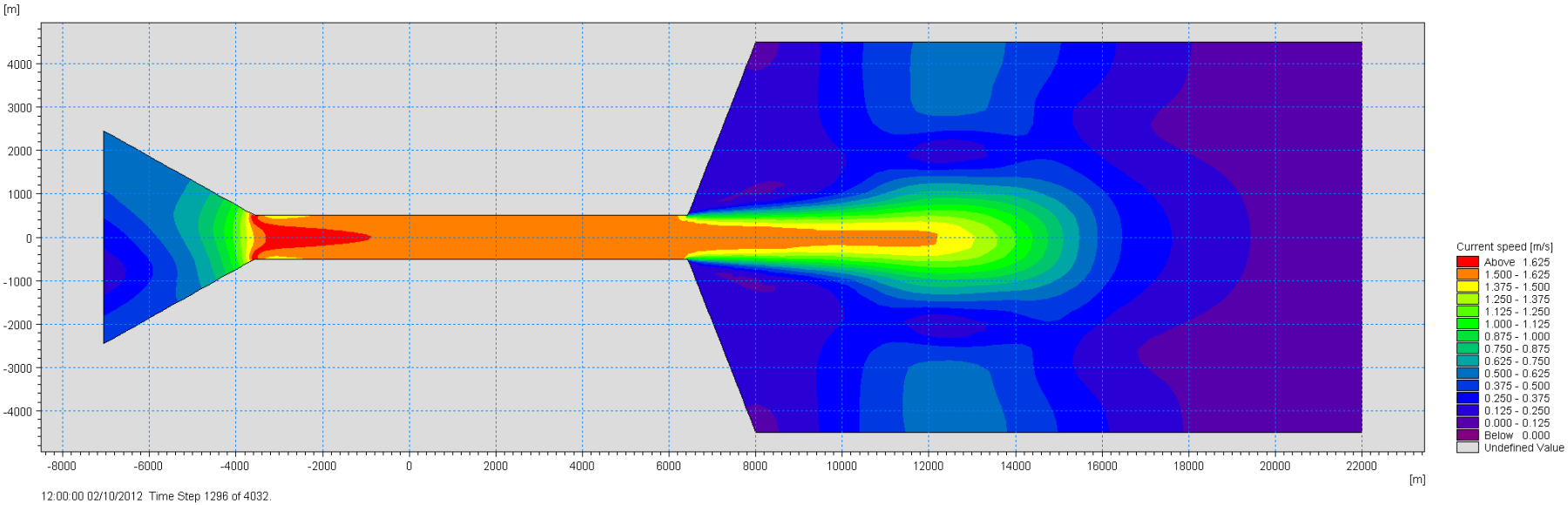


Idealised tidal basin based on Strangford Lough. Tidal forcing through periodic free surface elevation on the left open boundary. For now: fixed eddy viscosity ($10\ m^2/s$). Idealised bathymetry. Both models use the same triangular meshes.

Comparison in tidal basin - Flood

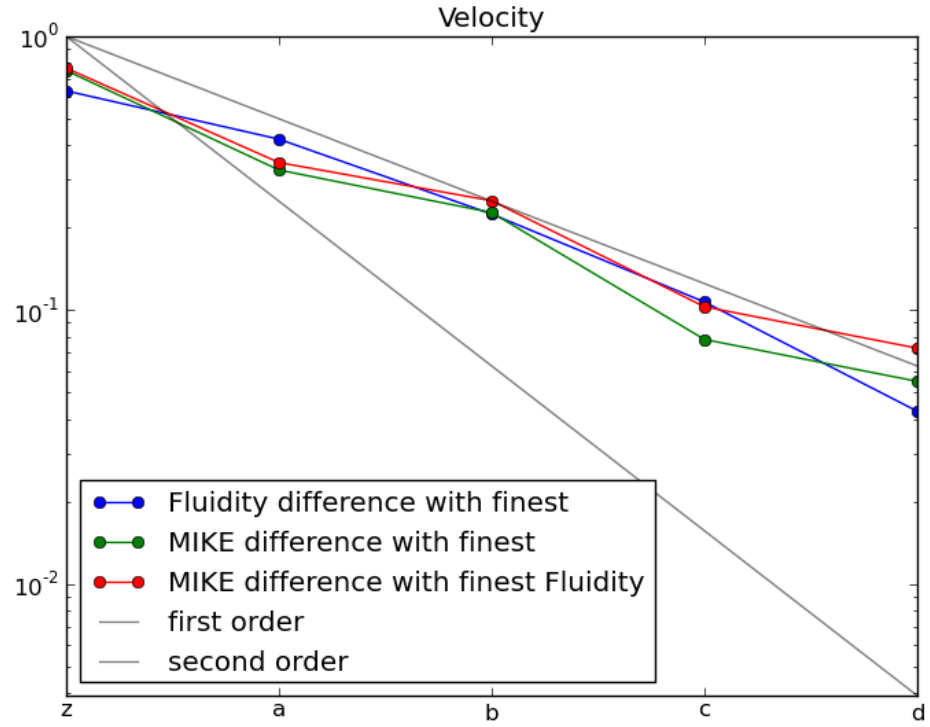


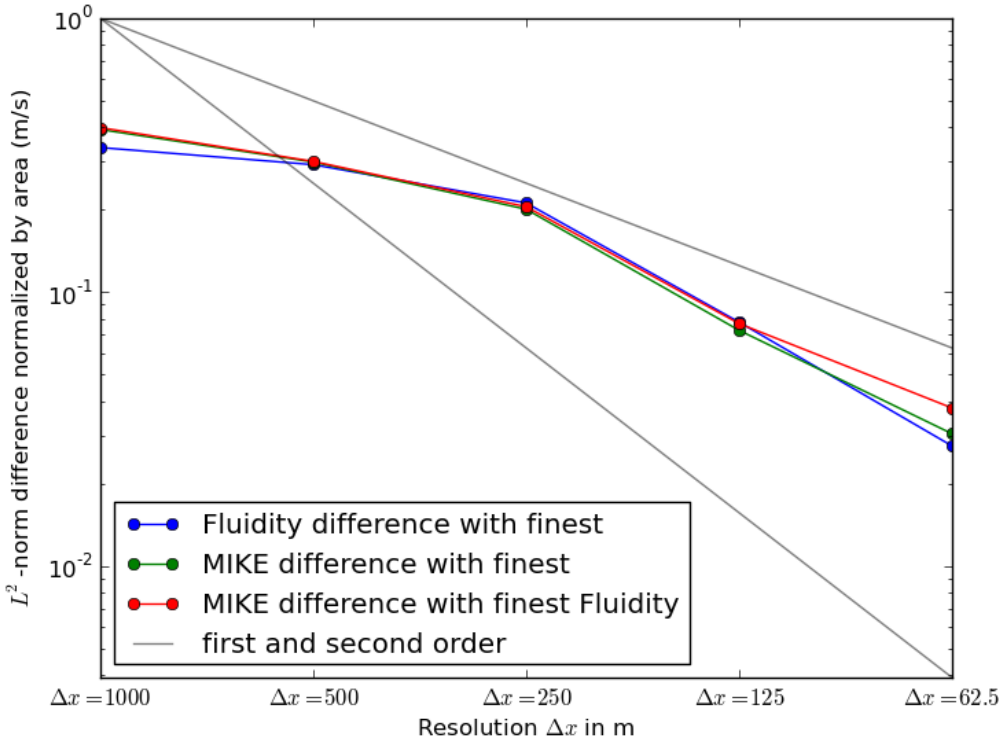
Fluidity - Shallow Water Equations



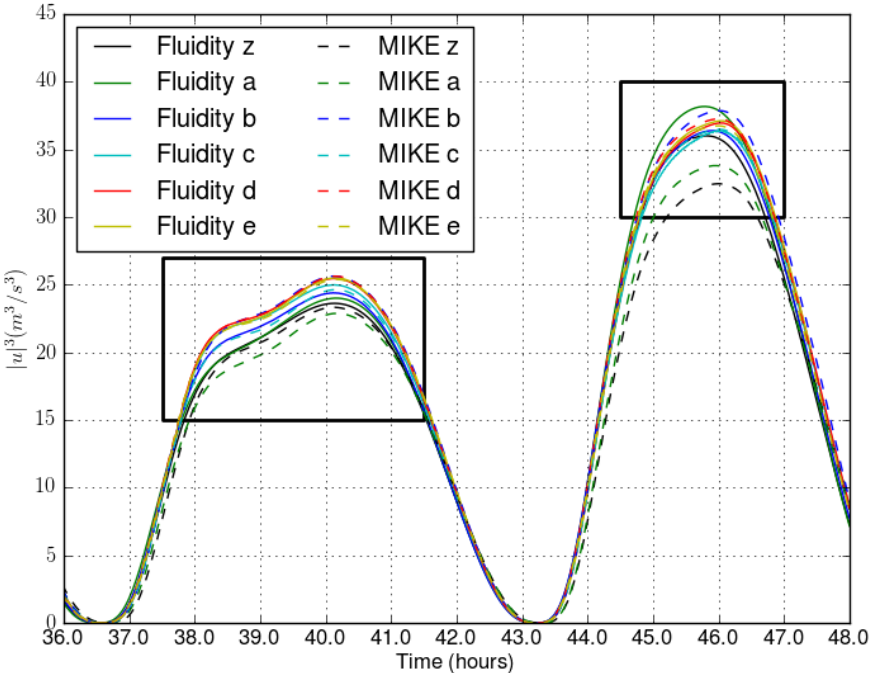
MIKE 21

Convergence of numerical results

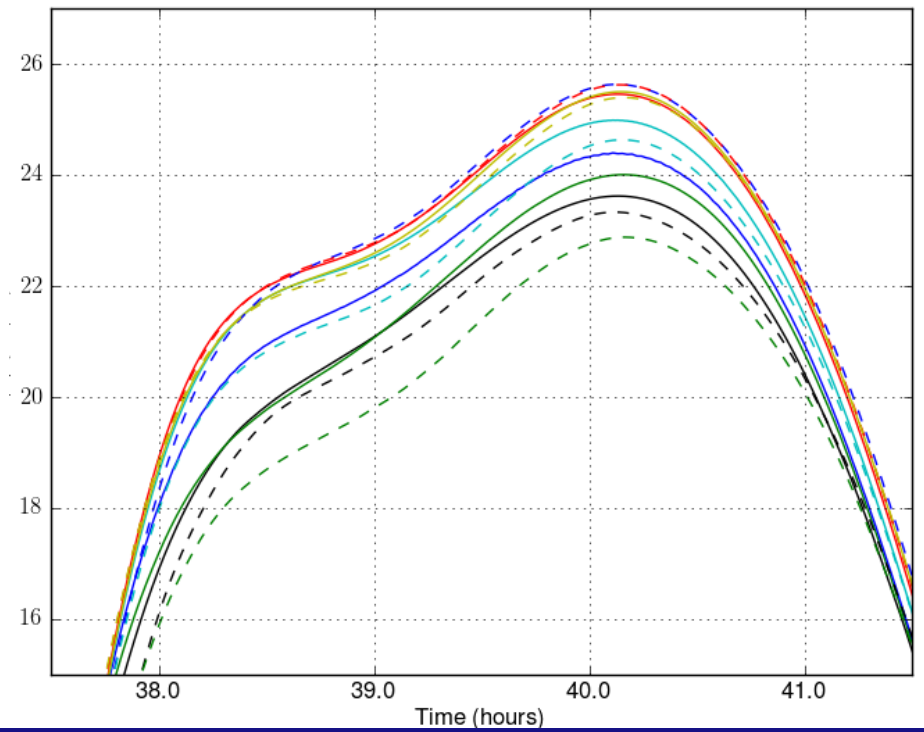


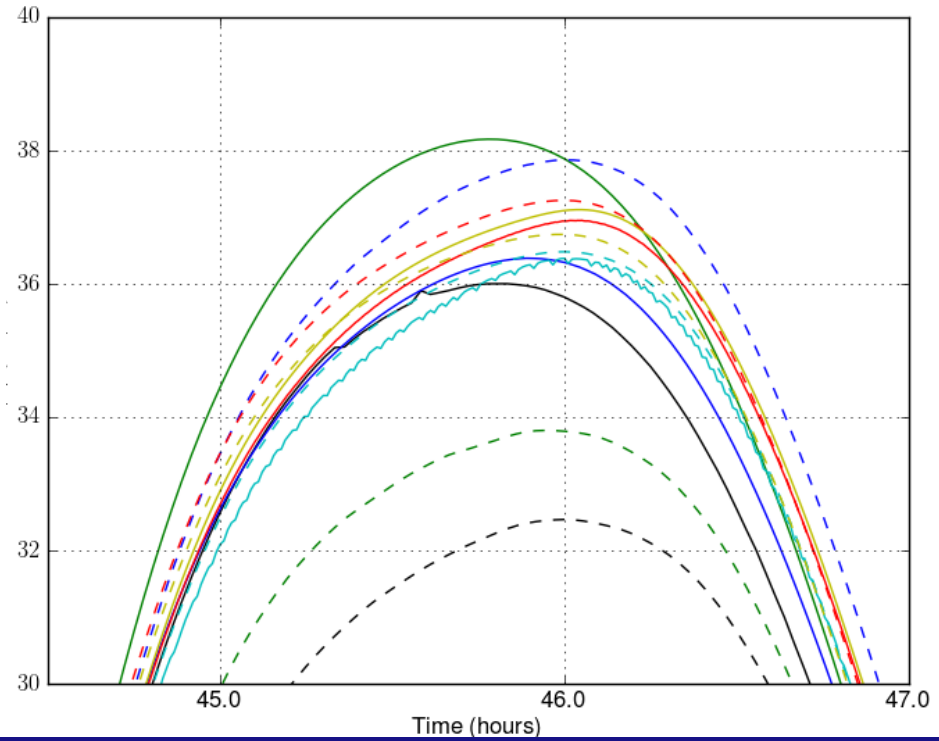


Integrated L^2-norm difference between model results at various mesh resolutions and at finest resolution ($\Delta x = 31.25$ m). Blue shows the convergence of Fluidity's results to its finest resolution result, and green the convergence of MIKE's results to its finest resolution. Red shows the convergence of MIKE's results to the finest resolution Fluidity results.

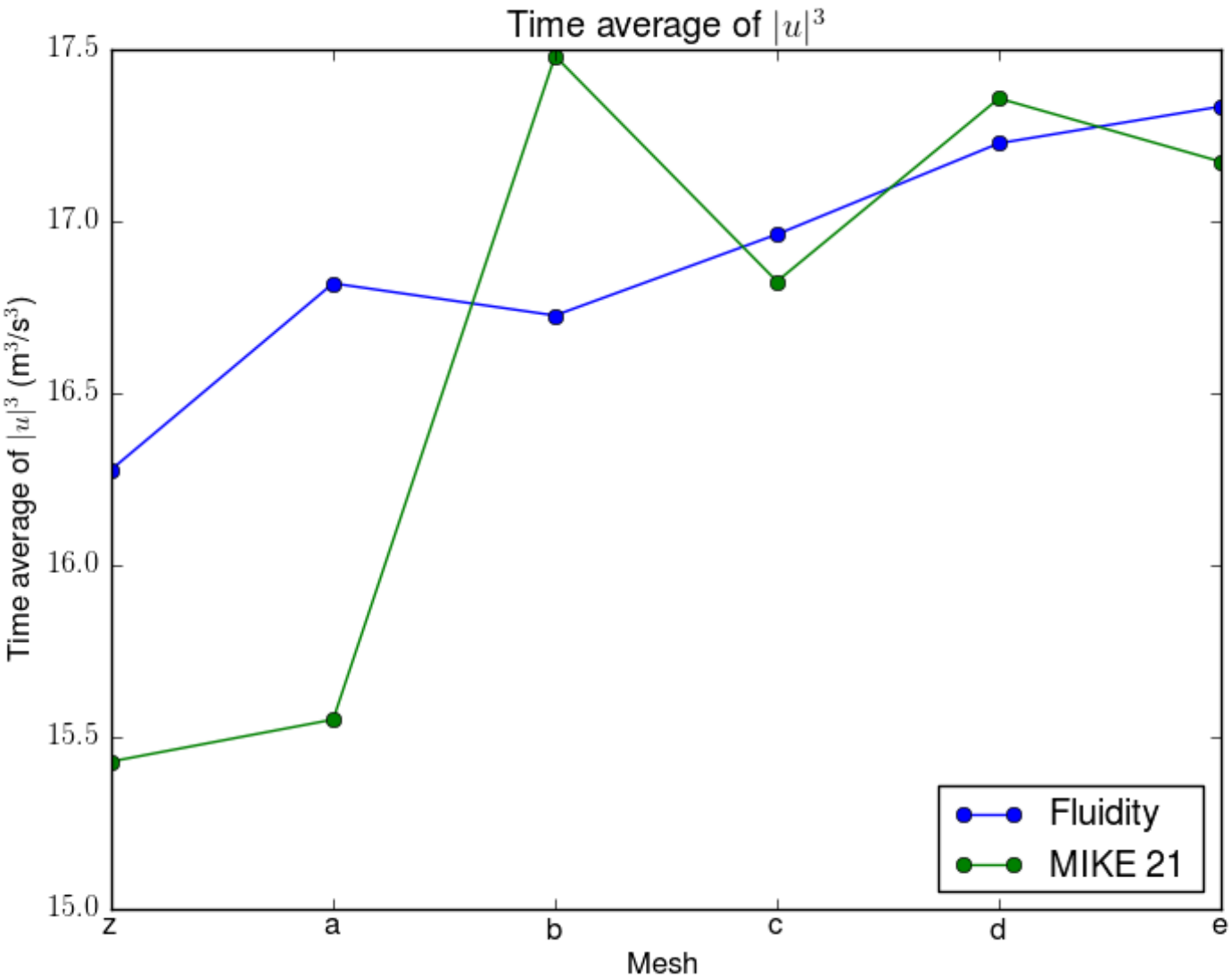


Velocity over time at a fixed point. Plotted here is u^3; This will determine energy yield of a turbine at this point.



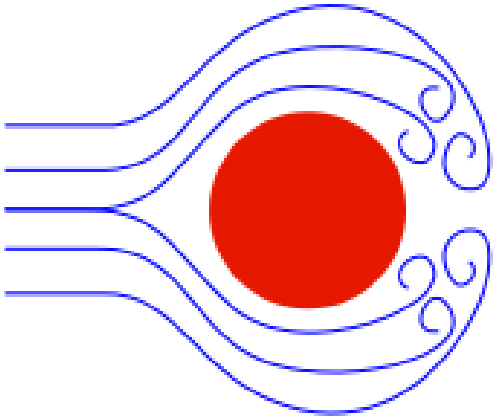


Temporal average



Convergence of temporal average of u^3 at a fixed point.

Parameterisation of turbines in SWE models



Total body drag force on turbine:

$$\vec{F}(\vec{u}) = \tfrac{1}{2}\rho_0 C_D(u) A_{\text{cross}} \|u\| \vec{u}$$

Represented as a momentum source $\vec{f}(\vec{u})$:

Depth avg. momentum eqn.: $\rho_0 \frac{\partial \vec{u}}{\partial t} + \rho_0 \vec{u} \cdot \vec{\nabla} \vec{u} + g \vec{\nabla} \eta + \cdots = \vec{f}(\vec{u})$

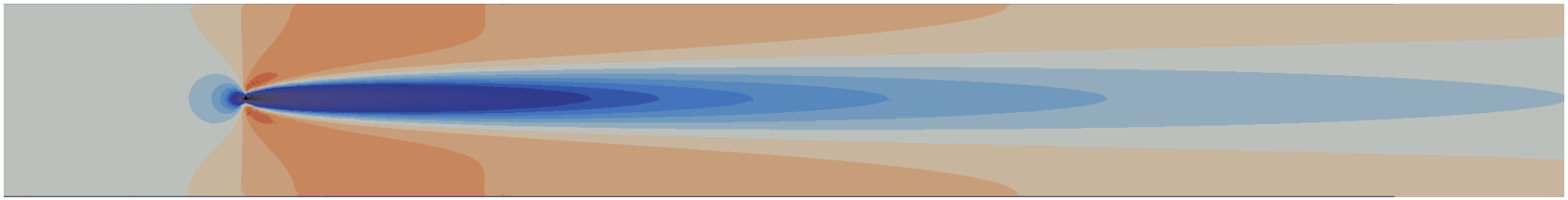
Applied over some horizontal area A, we need:

$$\int_A \vec{f}(\vec{u}) = \int_A \frac{\vec{F}(u)}{AH} = \int_A \frac{\rho_0 C_D(u) A_{\text{cross}}}{2AH} \|u\| \vec{u}$$

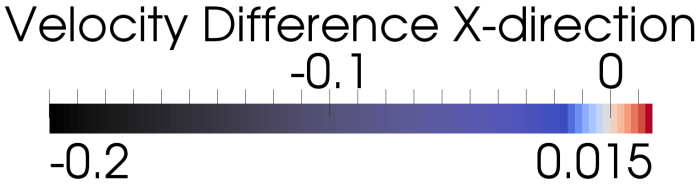
Therefore, we end up with a quadratic "bottom" drag force:

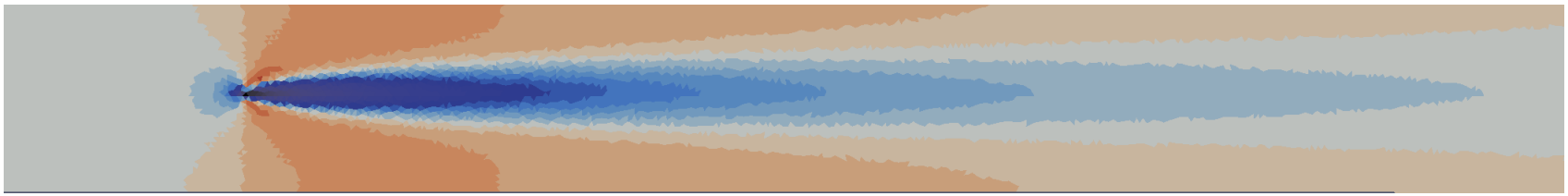
$$\vec{f}(\vec{u}) = \frac{\rho_0 C_D(u) A_{\text{cross}}}{2AH} \|u\| \vec{u}$$

Turbines in Fluidity and MIKE



Fluidity





MIKE 21

Visualisation of wake by velocity deficit (difference between solution with and without a turbine). Note that we do not expect a realistic wake as no turbulence model is used (fixed eddy viscosity).

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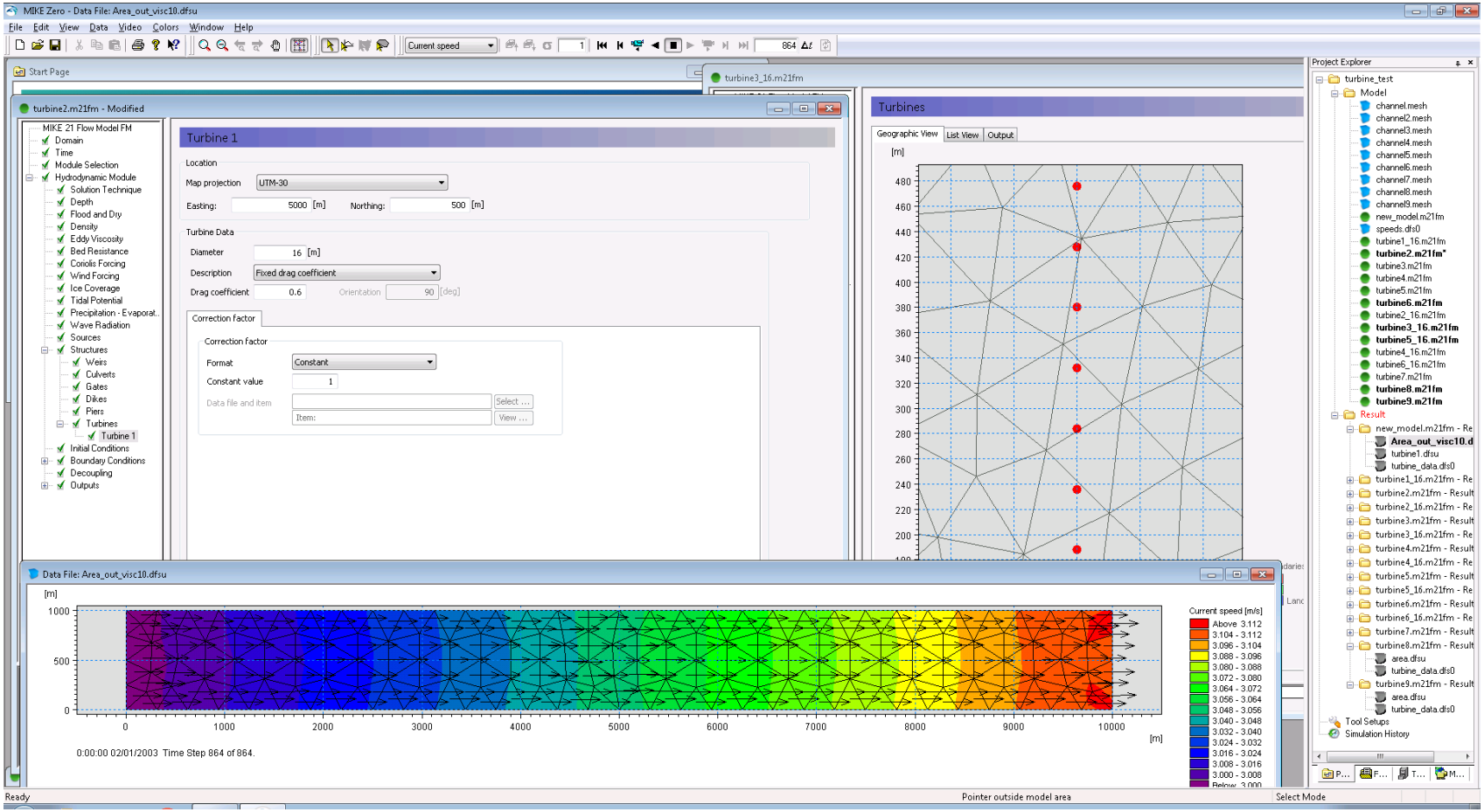
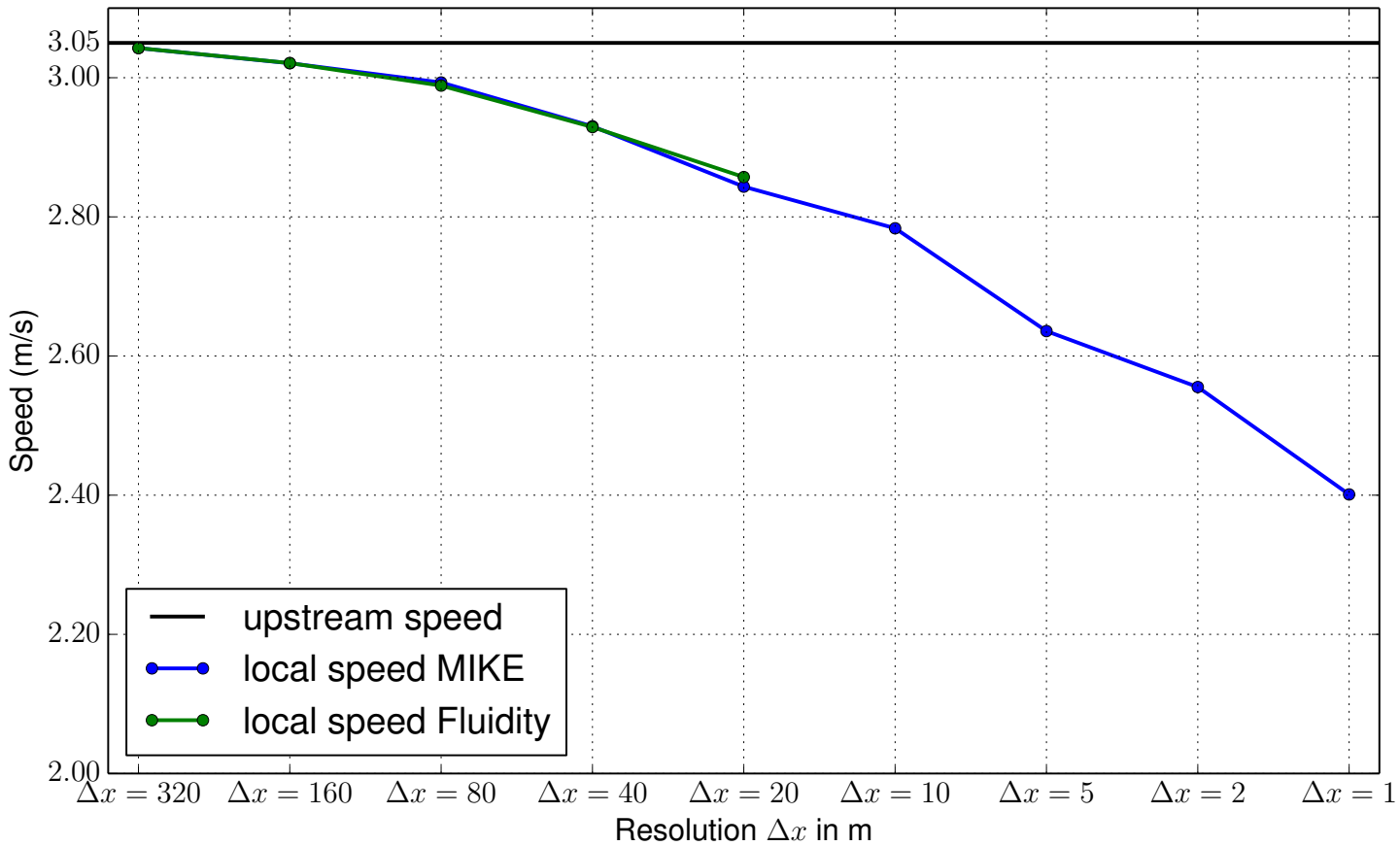
Turbines in Fluidity and MIKE



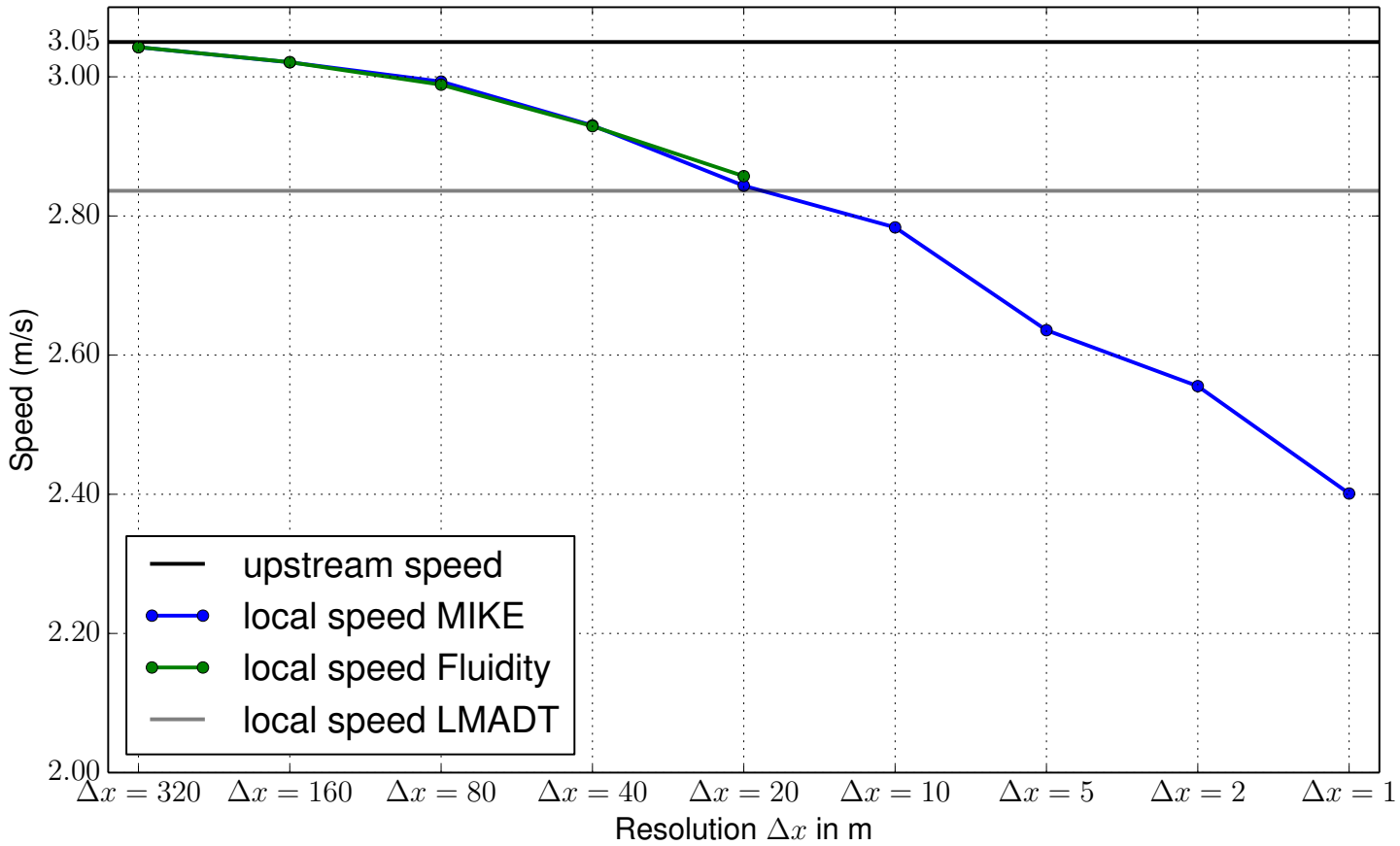
Turbines at coarse resolution are represented as point momentum sources. This means the finer the resolution, the smaller the area the drag is applied over. Leads to mesh dependency when resolving close to the turbine scale.

Drop in local velocity



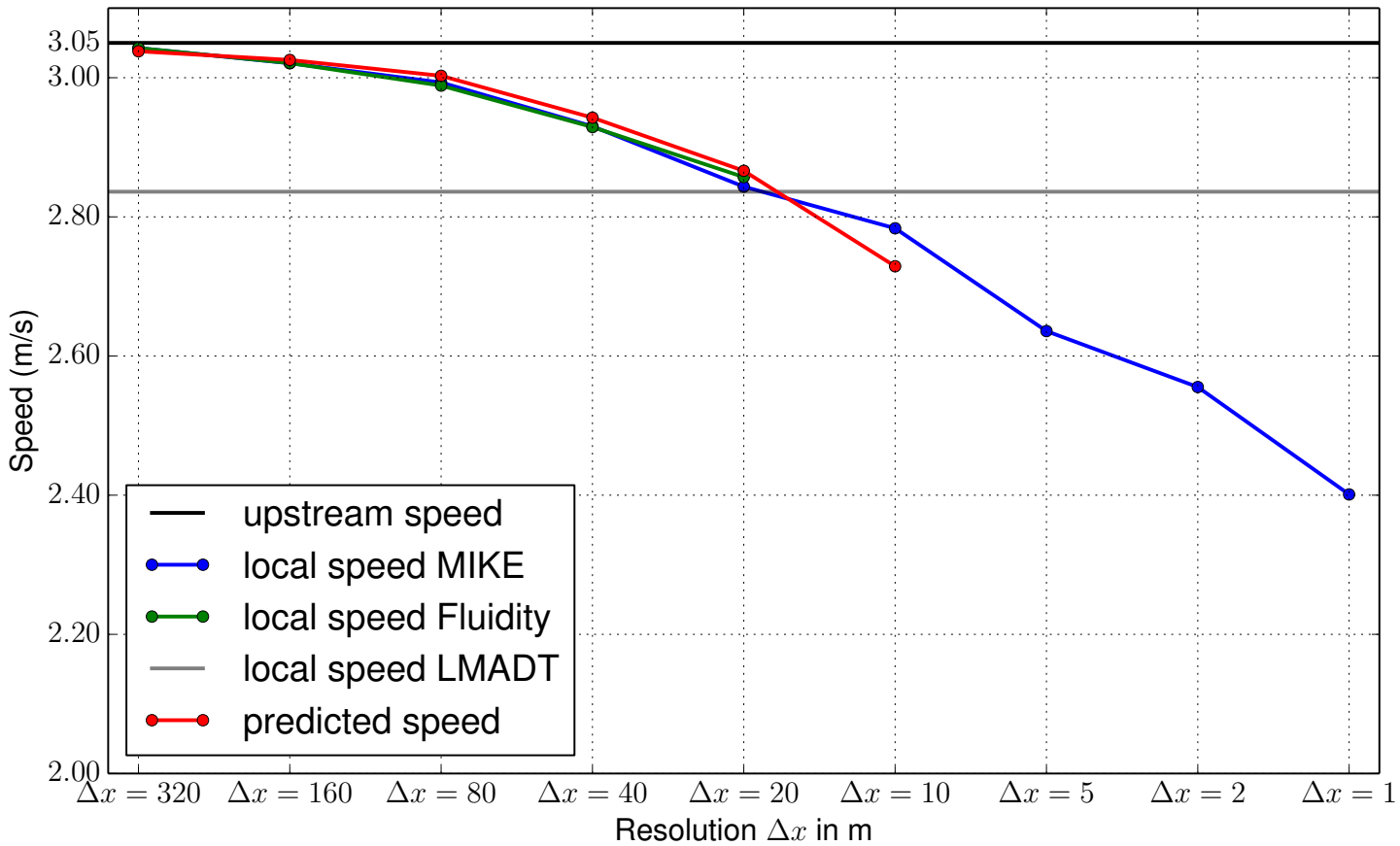
Local velocity in cell containing the turbine, drops significantly with increasing resolution. This local velocity is used to compute the drag force to be applied and the energy yield.

Drop in local velocity

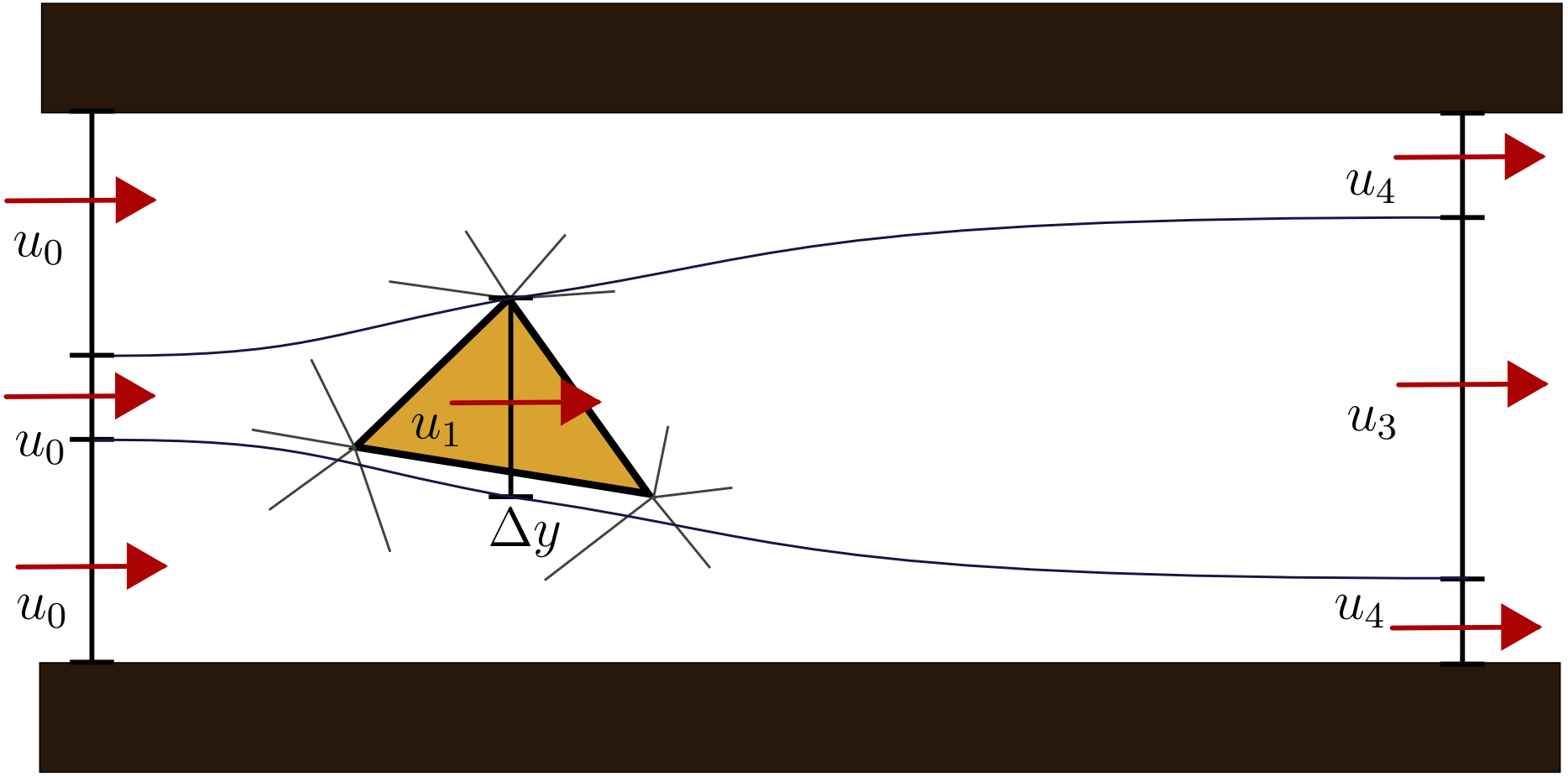


For resolution near the turbine scale (turbine diameter here is $D = 16$ m), the local velocity in cell matches the local turbine velocity predicted by actuator disc theory (LMADT).

Drop in local velocity



For resolutions larger than the turbine scale, the local velocity can be predicted by a modified actuator disc theory taking into account the length scale, Δx, over which the force is applied.



Adjust actuator disc theory (see Garret and Cummins '07), to take into account that the drag force $F = \frac{1}{2}\rho_0 C_D(u) A_{\text{cross}} u^2$, is not applied over the width of the turbine (diameter D), but over the width, Δy, of the cell the drag is applied within.

Correction in drag coefficient

Using the modified theory we can express the drag as a function of the local cell velocity instead of the upstream velocity:

$$F = \tfrac{1}{2}\rho_w C_T \frac{4}{\left(1+\sqrt{1-\gamma}\right)^2} A_{\text{cross}} \|u_{\text{local}}\|^2,$$

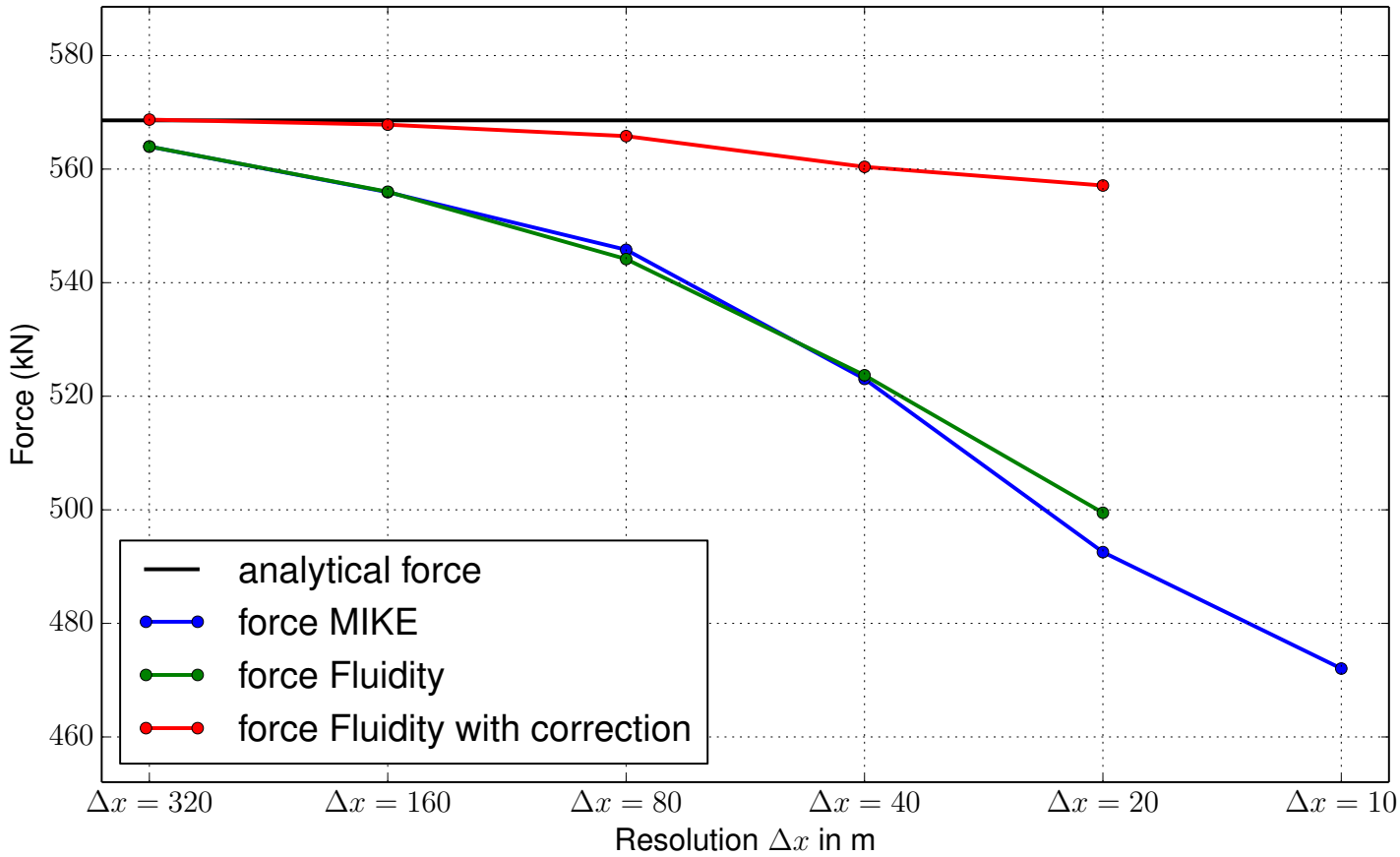
where

$$\gamma = C_T \frac{A_{\text{cross}}}{\Delta y H}.$$

Here $\Delta y H$ is the modified cross section, the product of the cell width Δy and water depth H.

In 3D, if we ignore the difference between the cell width and the turbine diameter, we have $\gamma = C_T$ and we recover the correction by Roc et al. '13. This assumes we resolve the turbine scale.

Correction in drag coefficient



Without correction, the drag force applied in the model drops significantly with resolution (up to 75 kN). With correction the change in applied force is limited (less than 12kN).

Conclusions:

- Study of hydrodynamic capabilities of MIKE 21 and Fluidity, for the tidal modelling of turbine farms with an application of energy resource and environmental impact assessments.
- Turbine drag parameterisation implemented as a point momentum source is mesh dependent and leads to an incorrect drag force for resolutions close to the turbine scale.
- Using actuator disc theory the drag coefficient can be corrected such that the correct force is applied.

Future work:

- Study capabilities for environmental impact studies: modelling biology with tracer fields, particles, and species interactions (ECO Lab).
- Improved wake representation through turbulence modelling. Comparison with 3D CFD and lab results.