Optimisation of tidal turbine array design and its impact upon performance

Dave Culley

image: Dave Culley

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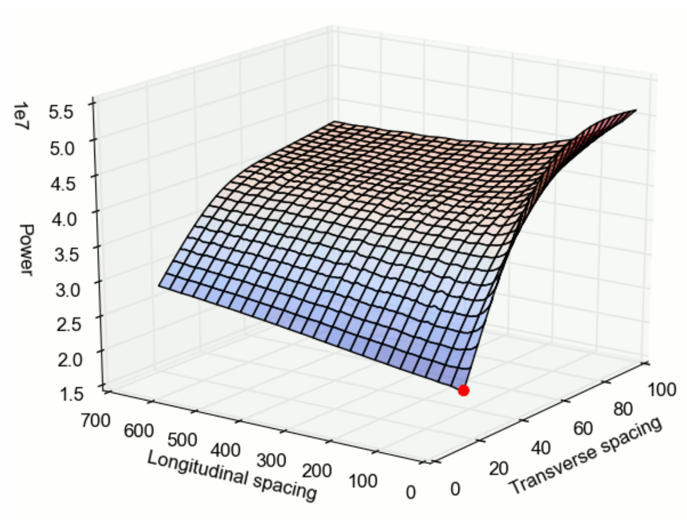
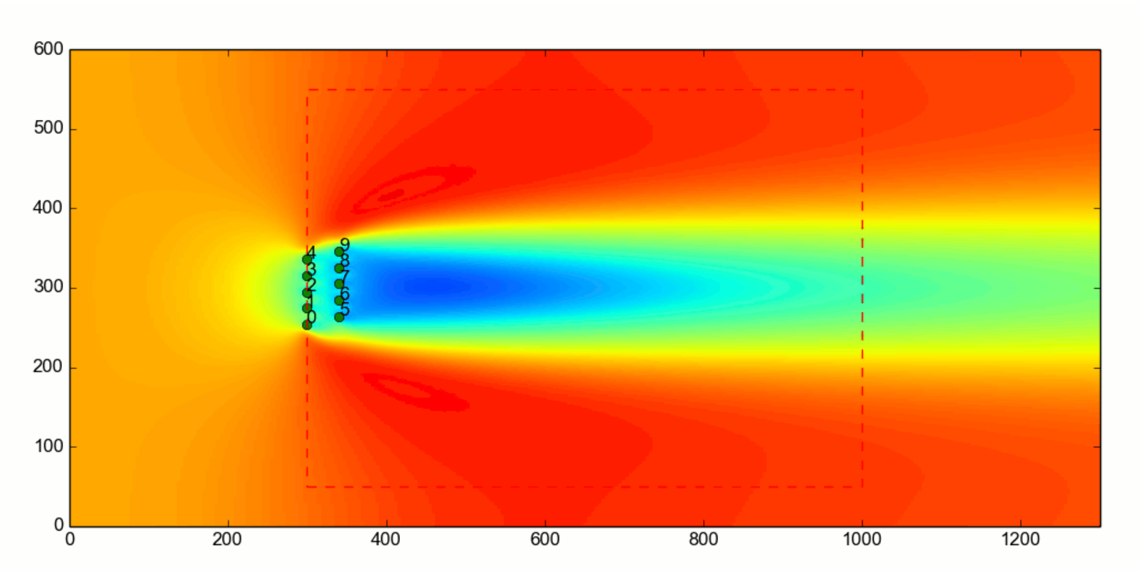
Motivation





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Micro-siting optimisation

By human intuition:

Manual approach based on intuition

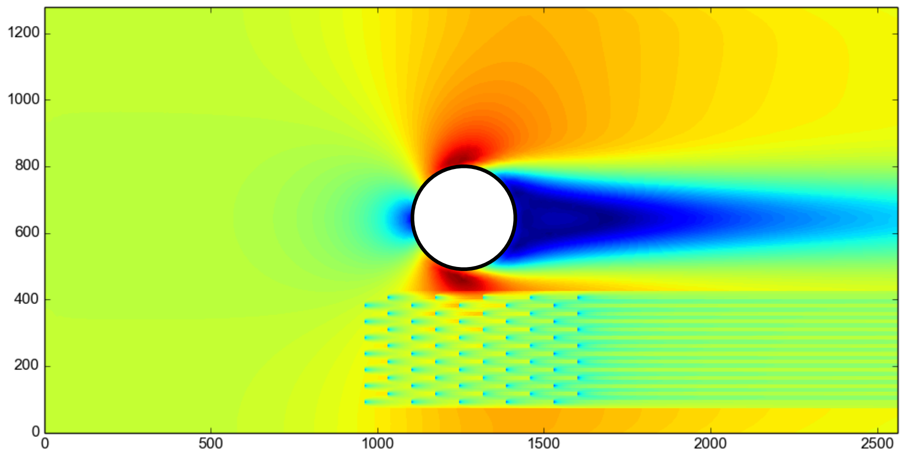
Without gradient data:

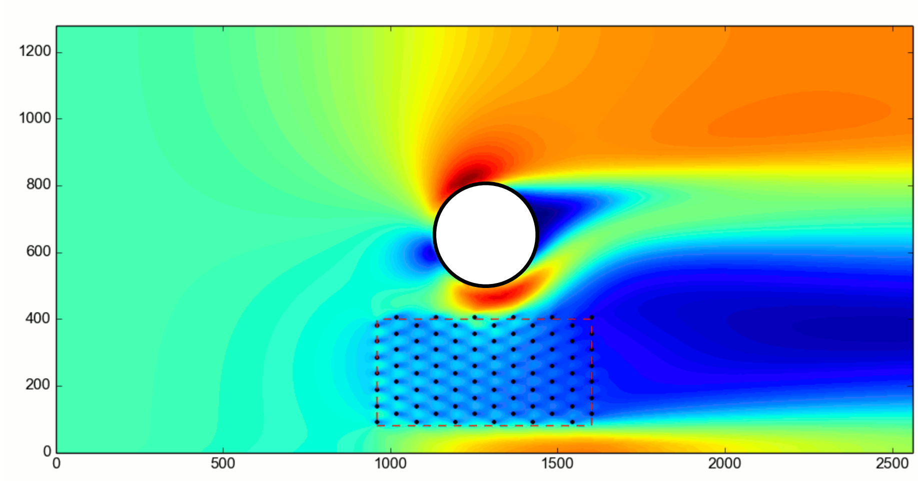
∴ limited to zero-order optimisation algorithms
∴ require large numbers of iterations ($\mathcal{O}$10^5 iterations?)
∴ use computationally cheap, uncoupled 'perturbation' approach
∴ physics of the problem is not well represented

OpenTidalFarm

OTF has access to the gradient of the objective **at a cost independent of the number of control parameters** (e.g. number of turbines)

Gradient based optimisation algorithms converge in orders of magnitude fewer iterations ($\mathcal{O}$100 iterations) than zero-order algorithms
∴ more computational budget for each solve
∴ can use more expensive (→ more accurate) flow models





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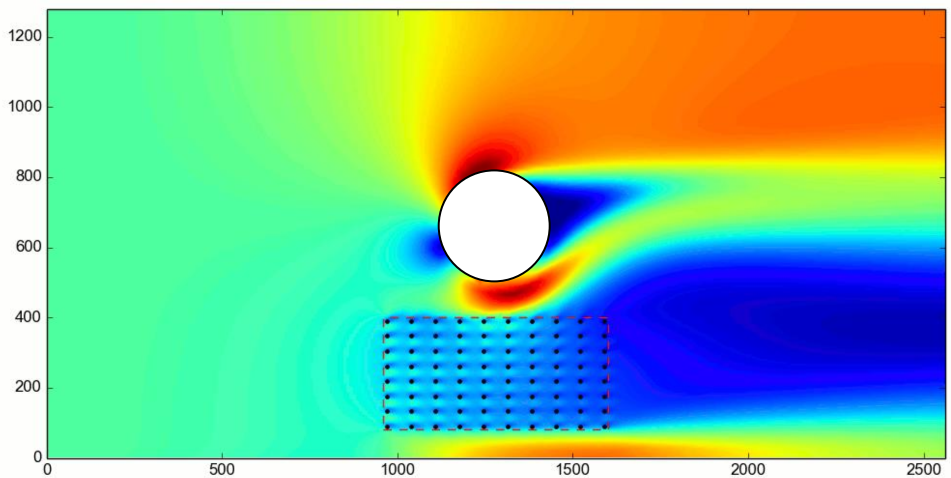

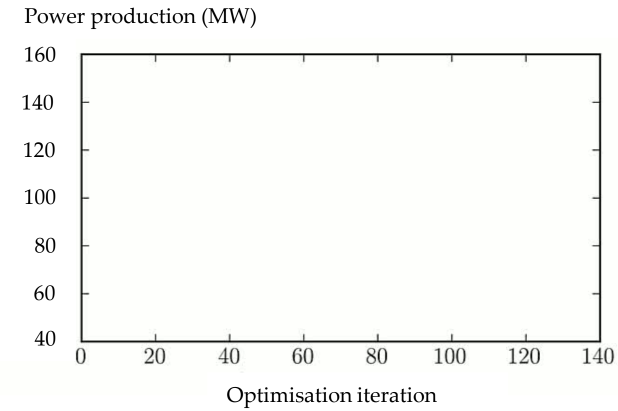

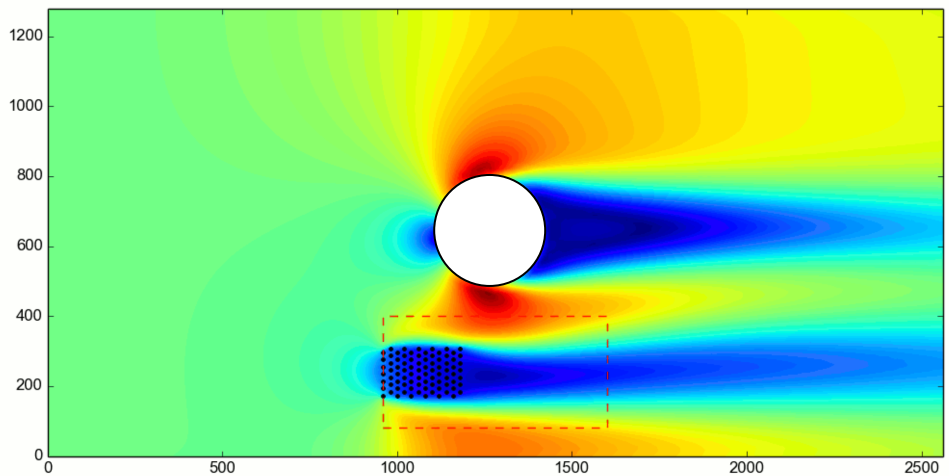

Complex domain geometry, bathymetry, turbine parametrisation etc.

Optimisation of micro-siting becomes non-trivial

Additional degrees of freedom -> improved solutions

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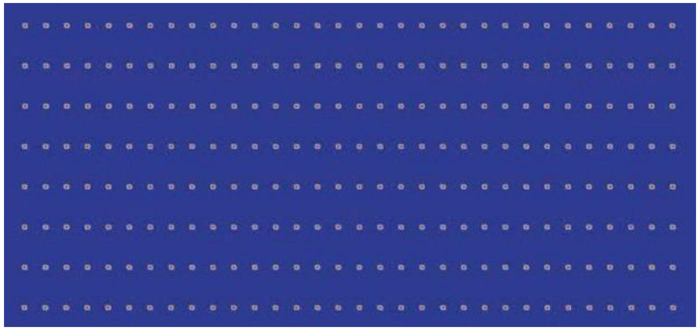


$$\max_{\mathbf{m}} \quad P(\mathbf{u},\mathbf{m})$$

$$\text{subject to} \quad F(\mathbf{u},\mathbf{m}) = 0$$

SW equations must be satisfied

$$\mathbf{b}_l \leq \mathbf{m} \leq \mathbf{b}_u$$

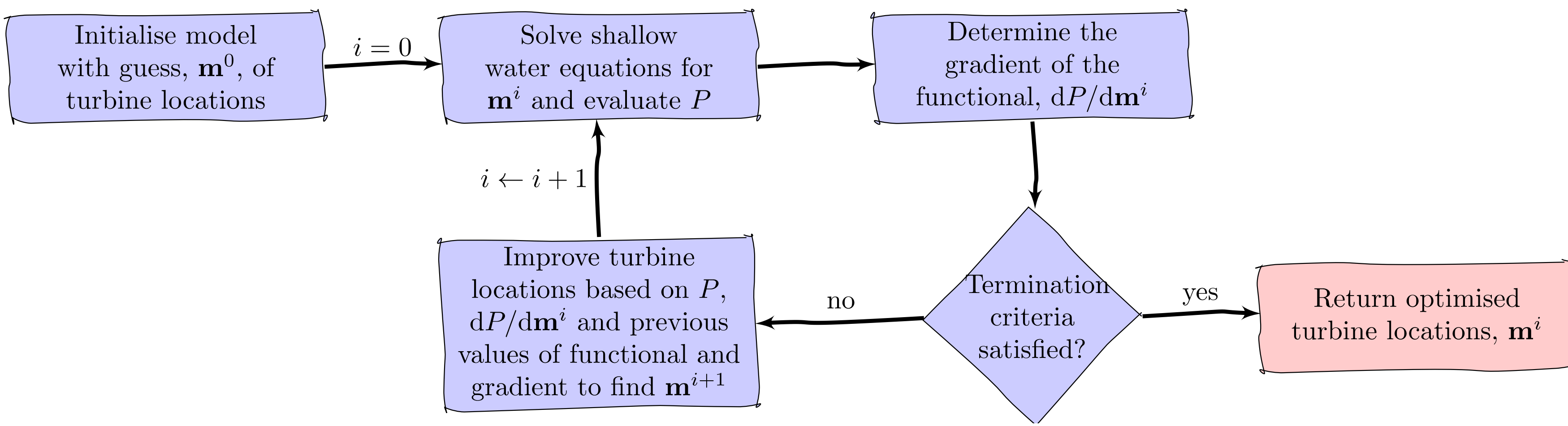
turbines must remain within a given bounded turbine area

$$g(\mathbf{m}) \leq 0.$$

turbines must not move too close to one another

encode the turbines in the vector **m**

$$\mathbf{m} = (x_1, y_1, x_2, y_2, \ldots, x_n, y_n)^T.$$



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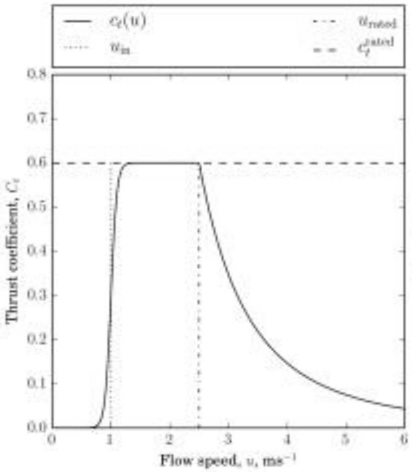
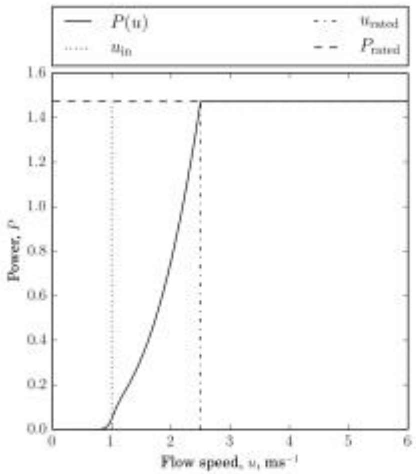
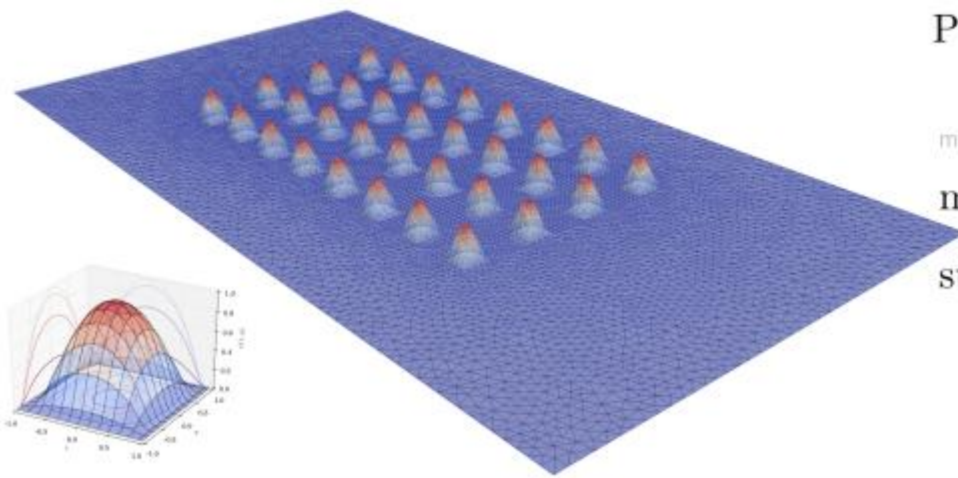
Turbine parametrisation



Thrust coefficient



Power curve



u is the solution to the non-linear shallow water equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\nabla^2\mathbf{u} + g\nabla\eta + \frac{c_b + c_t}{H}\|\mathbf{u}\|\mathbf{u} = 0$$

$$\frac{\partial \eta}{\partial t} + \nabla\cdot(H\mathbf{u}) = 0$$

encode the turbines in the vector **m**

$$\mathbf{m} = (x_1, y_1, x_2, y_2, \ldots, x_n, y_n)^T.$$

total power is integral over domain of power extracted due to turbine drag

$$\mathrm{P}(\mathbf{m}) = \int_\Omega \rho c_t(\mathbf{m},\mathbf{u})||\mathbf{u}||^3 \,\mathrm{d}x.$$

maximise the power through manipulating **m**

$$\begin{aligned} \max_{\mathbf{m}} \quad & P(\mathbf{u},\mathbf{m}) \\ \text{subject to} \quad & F(\mathbf{u},\mathbf{m}) = 0 \\ & \mathbf{b}_l \le \mathbf{m} \le \mathbf{b}_u \\ & g(\mathbf{m}) \le 0. \end{aligned}$$

SW equations must be satisfied

turbines must remain within a given bounded turbine area

turbines must not move too close to one another

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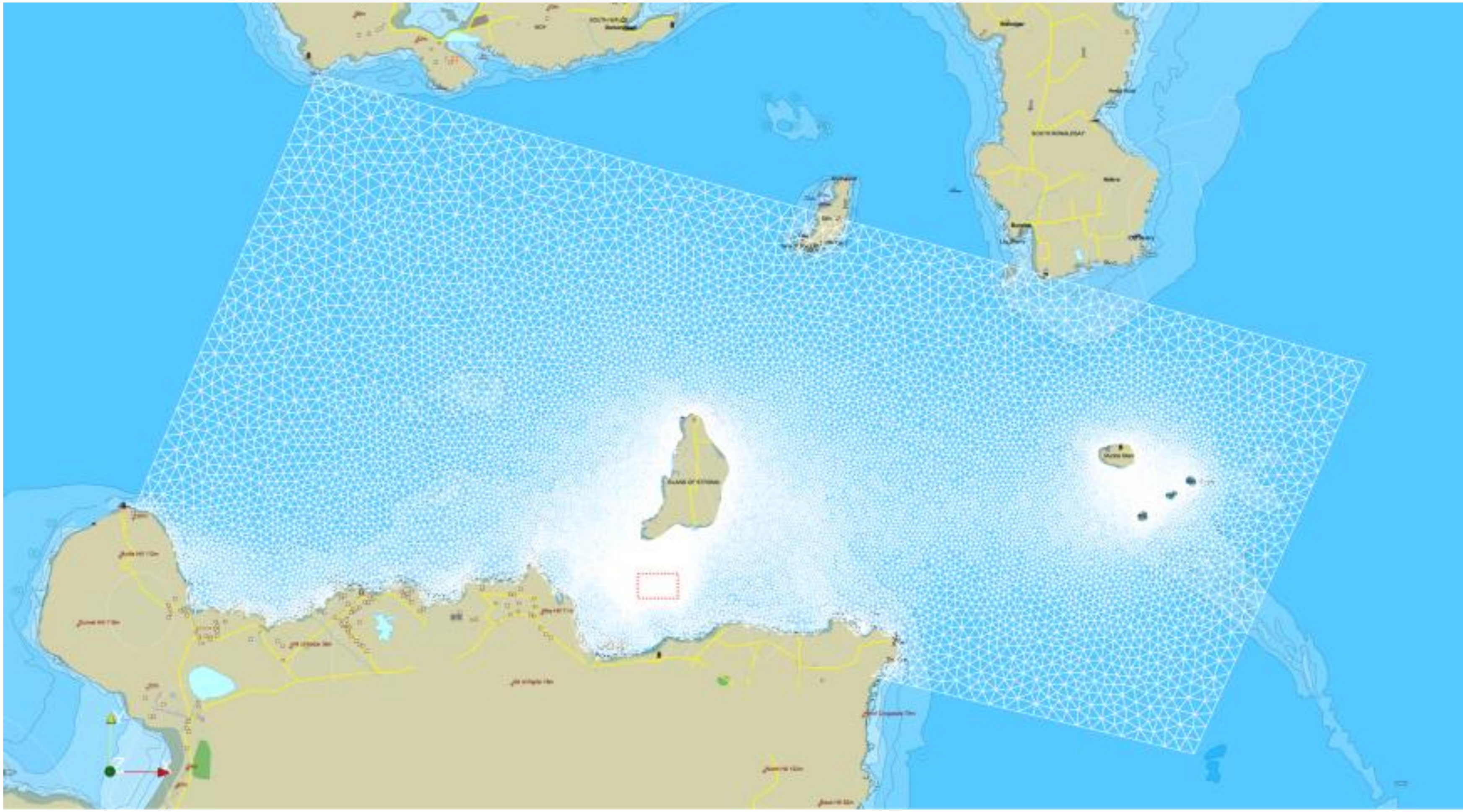

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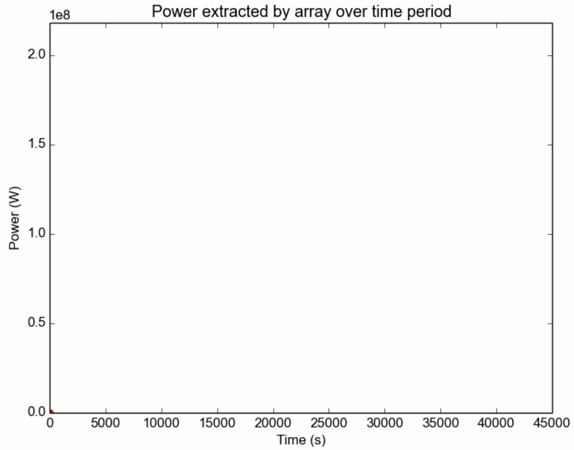
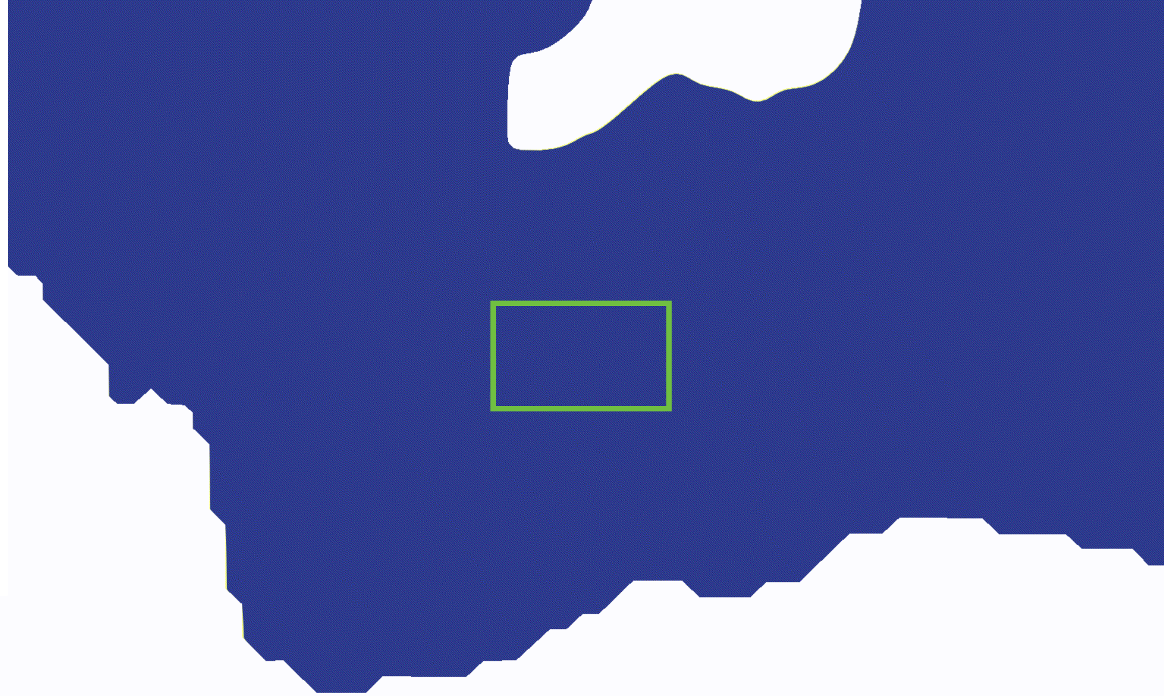
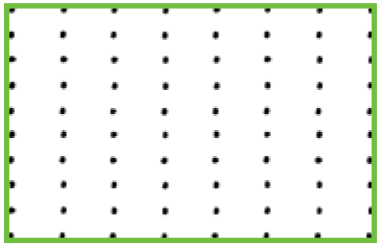
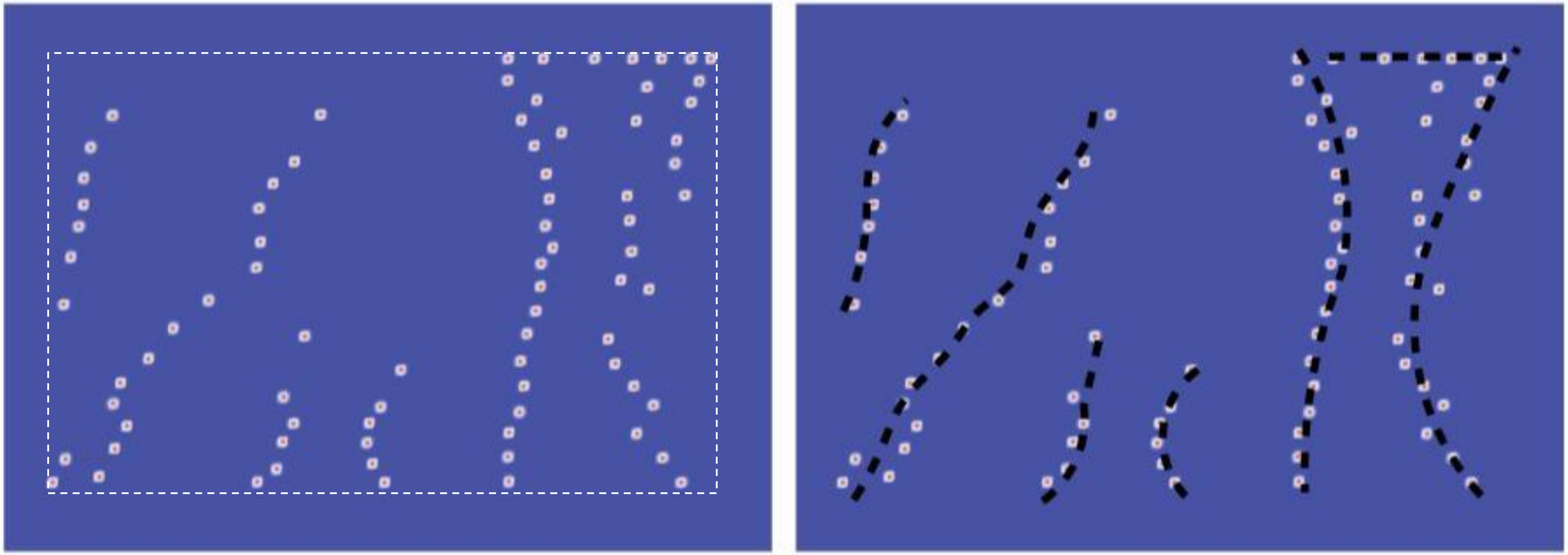

Power extracted by array over time period
1e8
2.0
1.5
1.0
0.5
0.0
Power (W)
0
5000
10000
15000
20000
25000
30000
35000
40000
45000
Time (s)



Proposed layout





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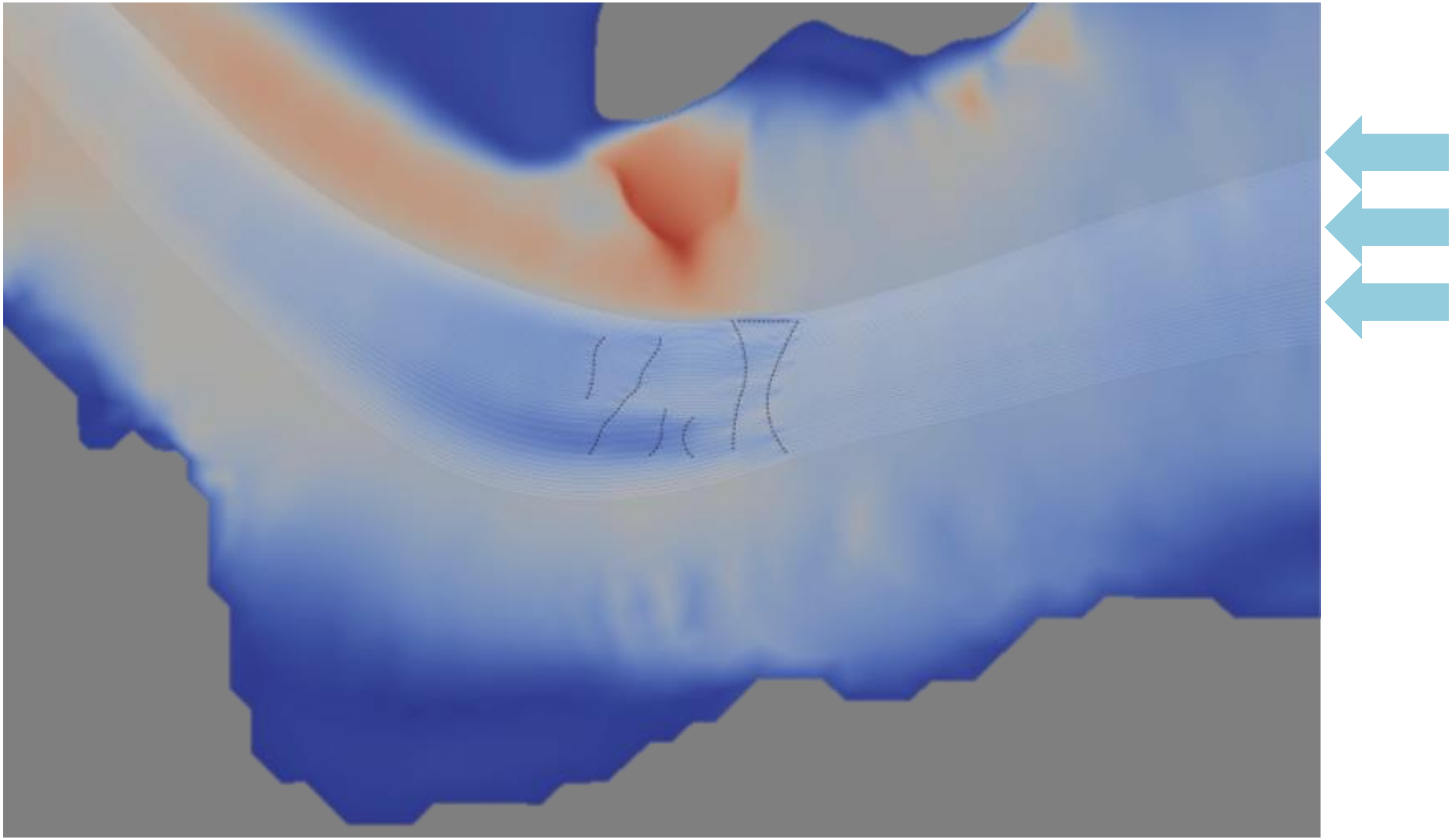
Proposed layout



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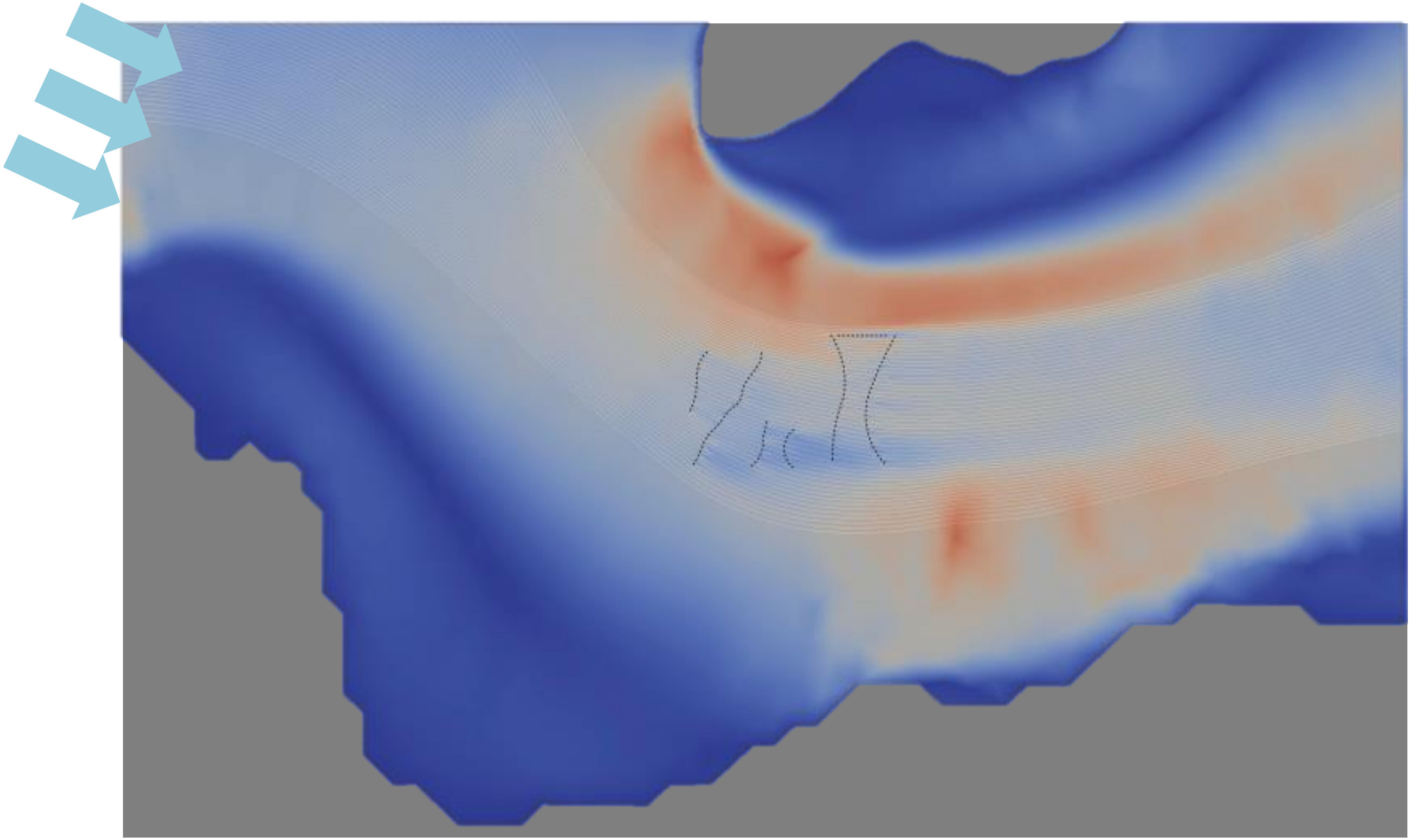
Proposed layout



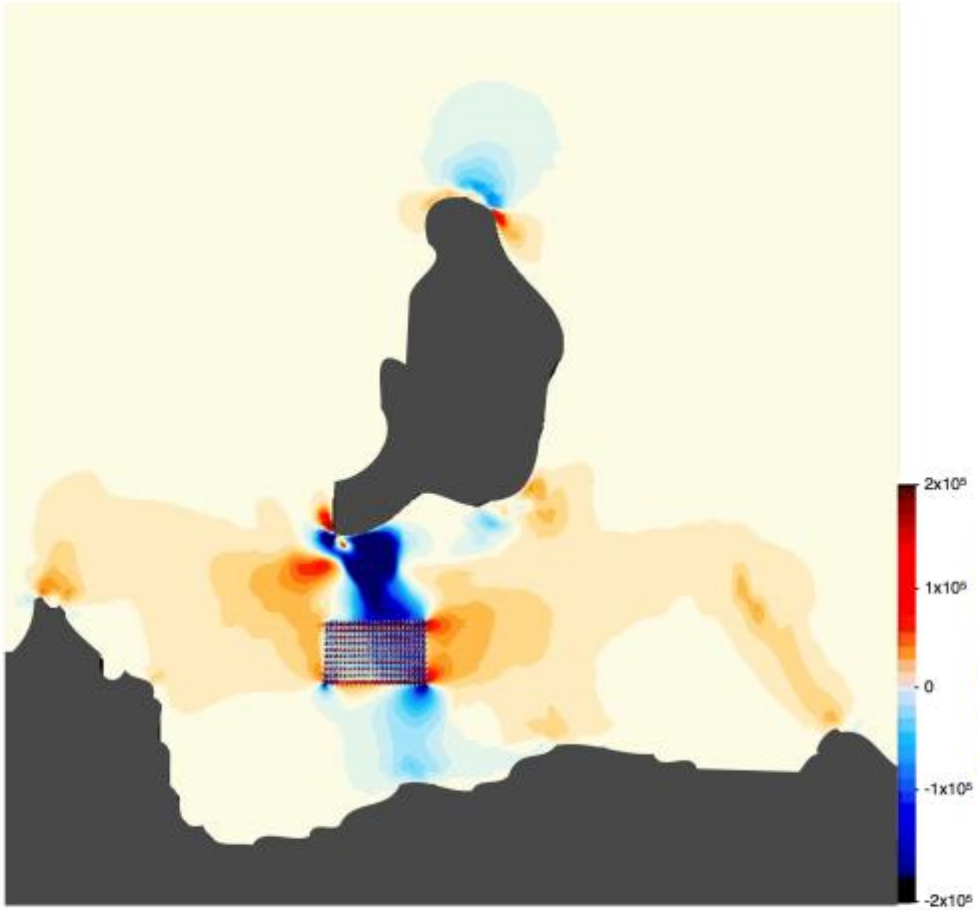
Sensitivity analysis

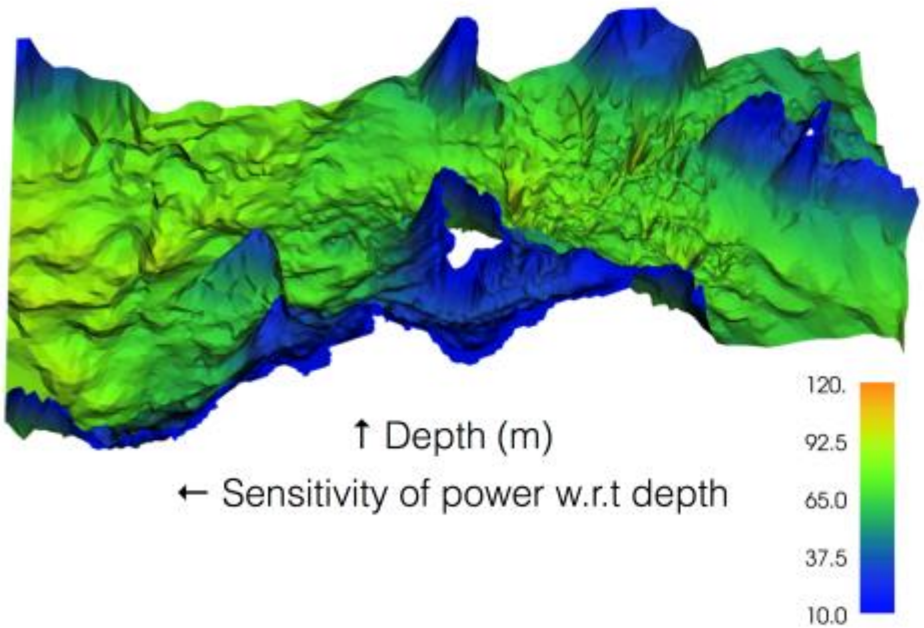
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\nabla^2\mathbf{u} + g\nabla\eta + \frac{c_b + c_t}{H}\|\mathbf{u}\|\mathbf{u} = 0$$

$$\frac{\partial \eta}{\partial t} + \nabla\cdot(H\mathbf{u}) = 0$$

$$\mathrm{P}(\mathbf{m}) = \int_\Omega \rho c_t(\mathbf{m},\mathbf{u})||\mathbf{u}||^3\,\mathrm{d}x.$$

$$\frac{\mathrm{d}P}{\mathrm{d}\mathbf{m}} \qquad \frac{\mathrm{d}P}{\mathrm{d}H} \qquad \frac{\mathrm{d}P}{\mathrm{d}c_b}$$





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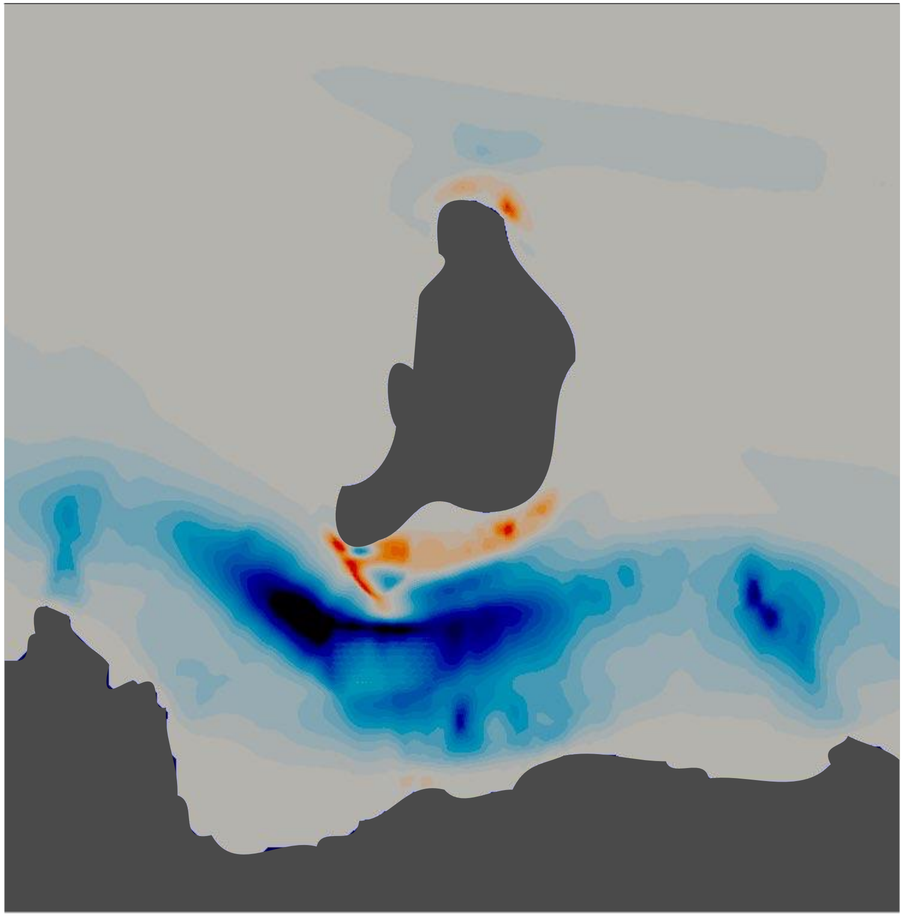
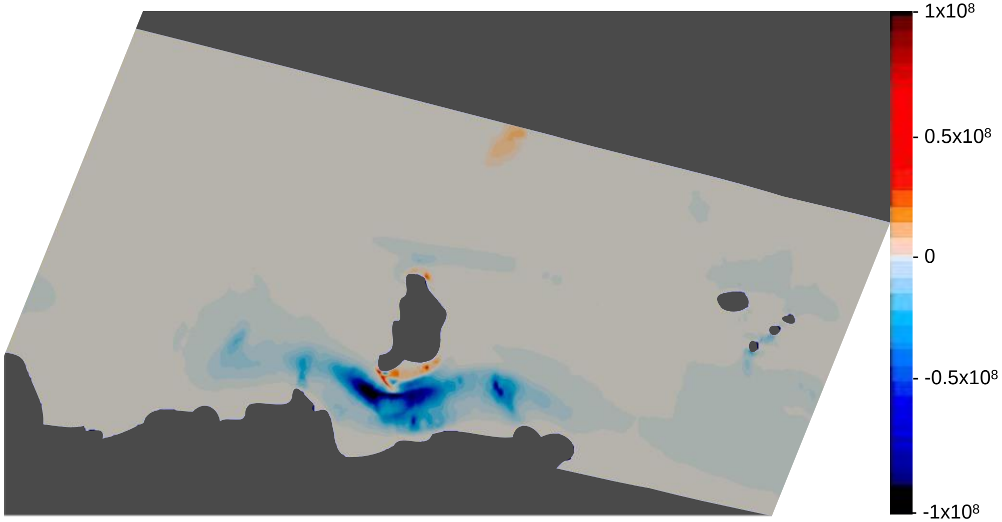
Sensitivity analysis





Sensitivity of power with respect to bottom friction (constant 0.0025 over domain)

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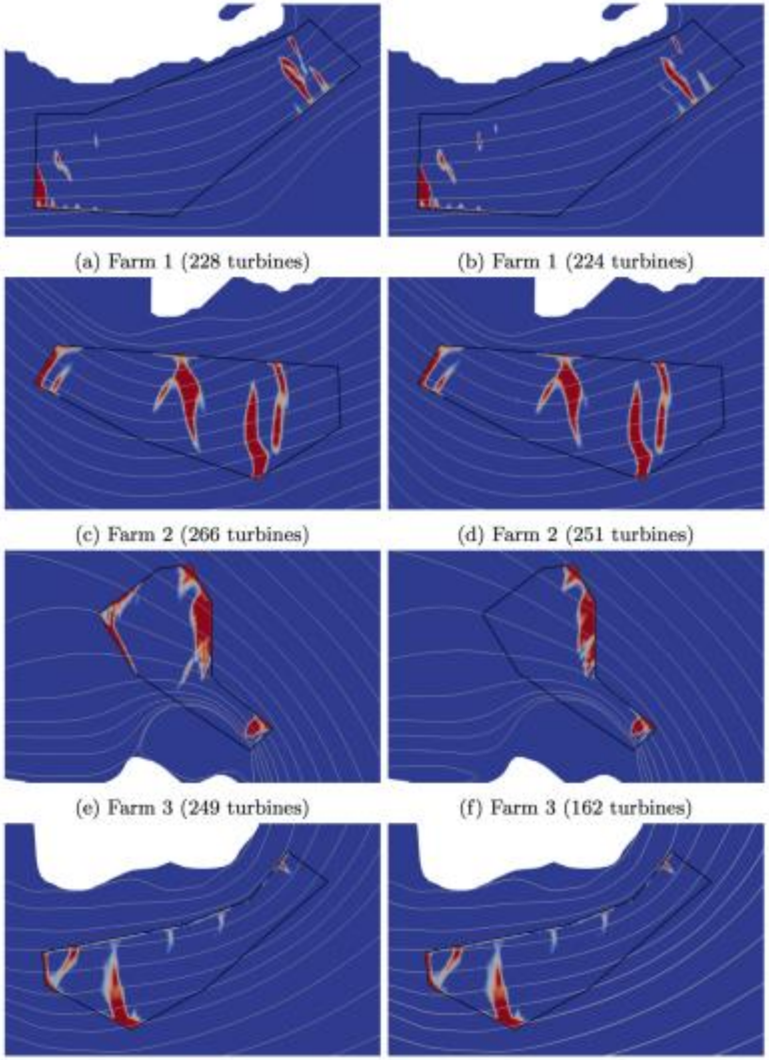
An alternate turbine parametrisation



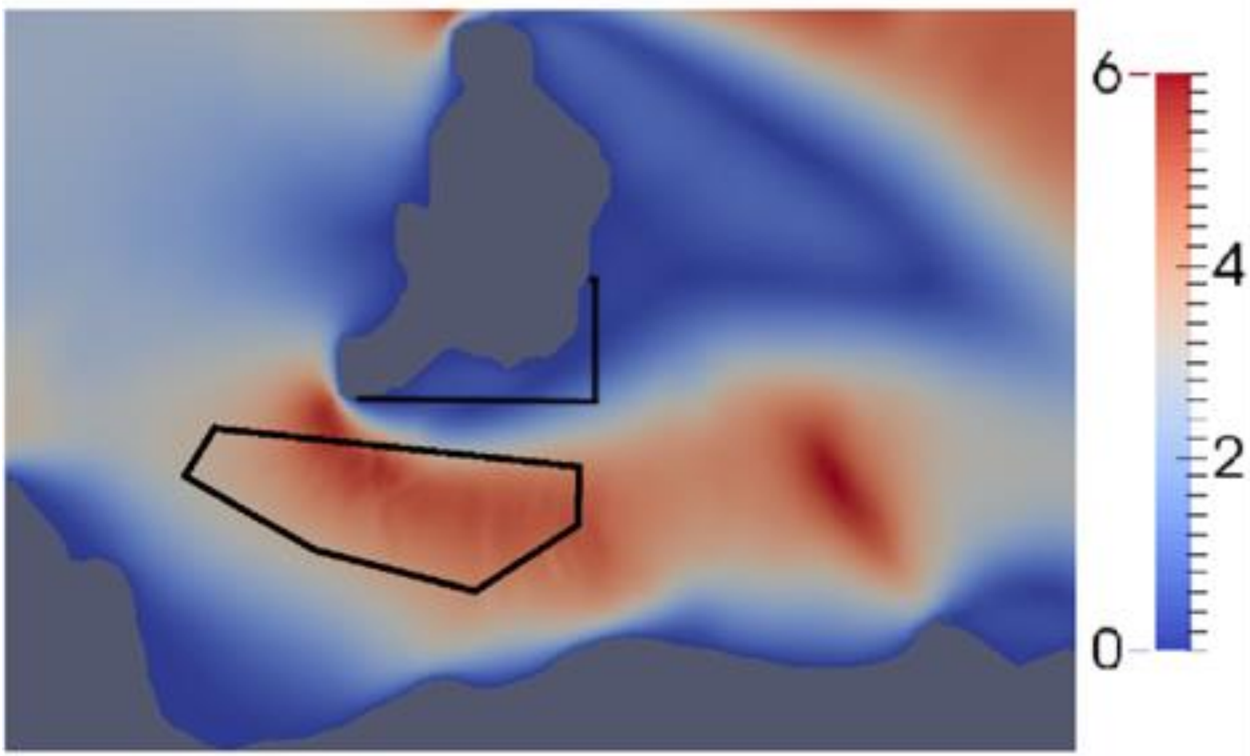
(a) Farm 1 (228 turbines) (b) Farm 1 (224 turbines)
(c) Farm 2 (266 turbines) (d) Farm 2 (251 turbines)
(e) Farm 3 (249 turbines) (f) Farm 3 (162 turbines)
(g) Farm 4 (156 turbines) (h) Farm 4 (153 turbines)

Funke, Kramer, Piggott. *Design optimisation and resource assessment for tidal-stream renewable energy farms using a new continuous turbine approach*. 2016

Implementing environmental constraints

Areas of the domain can be protected from change in the flow regime in order, for example, to preserve valuable coastline or habitat.

A *preservation domain* is drawn, and a *preservation weight* determines how important it is that that area is protected. The higher the weight, the greater the protection.



Peak flood flow (m/s).

Farm site (below) and preservation domain (above) are outlined in black.

du Feu, et al. *The trade off between tidal-turbine array yield and environmental impact; a multi-objective optimisation problem.* 2016

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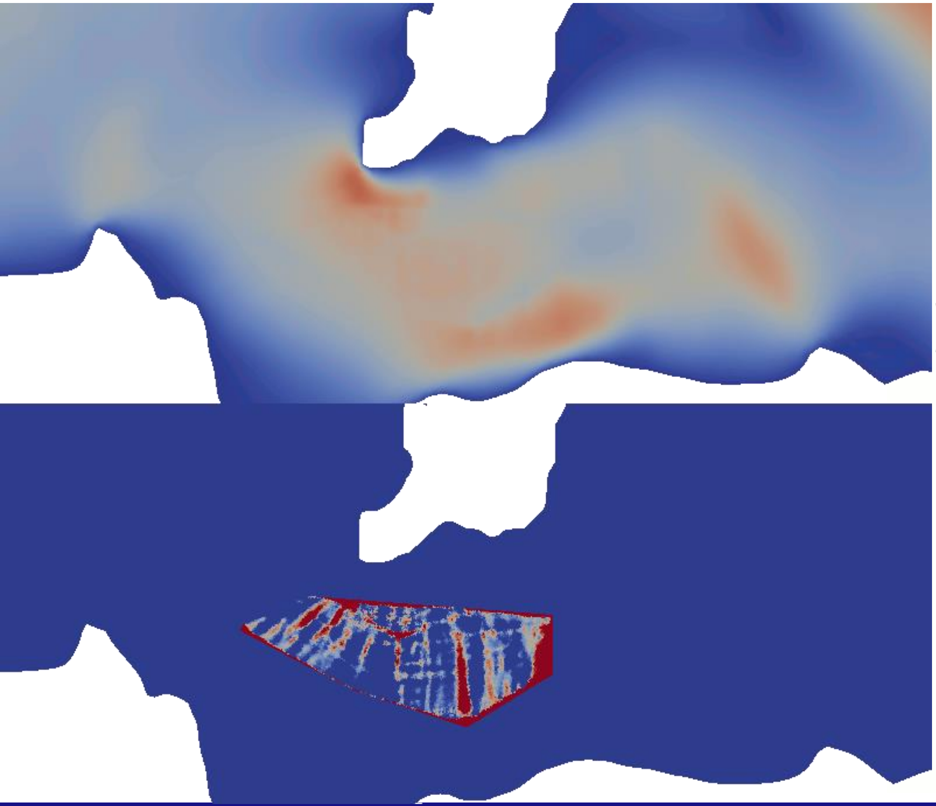
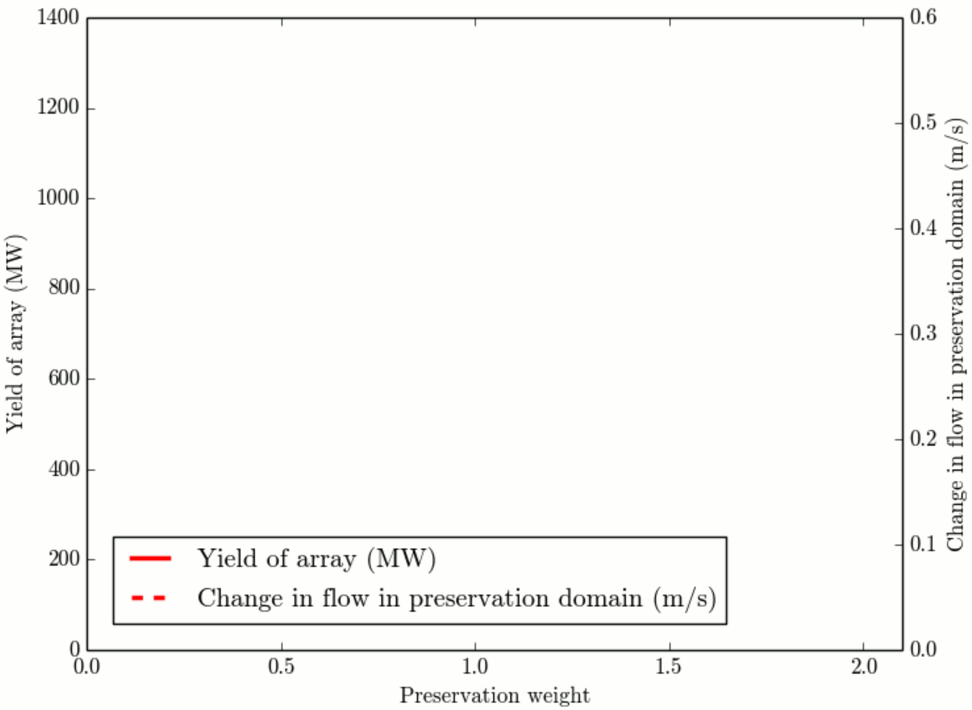
Implementing environmental constraints

As the preservation weight is increased the turbines move upstream away from the preservation domain, and the total number of turbines is reduced.

Velocities within the preservation domain then return towards their ambient level, with the compromise of a loss in array yield.





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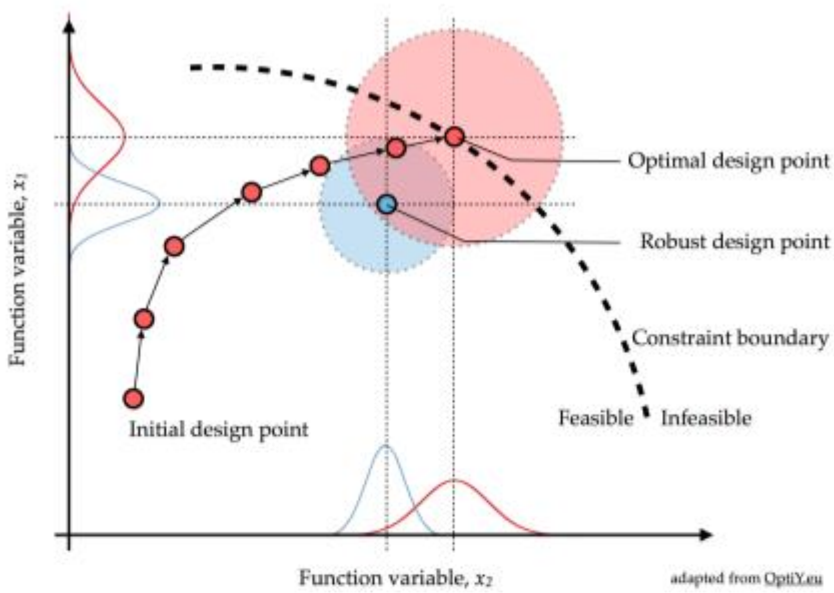
Future work

Robust optimisation



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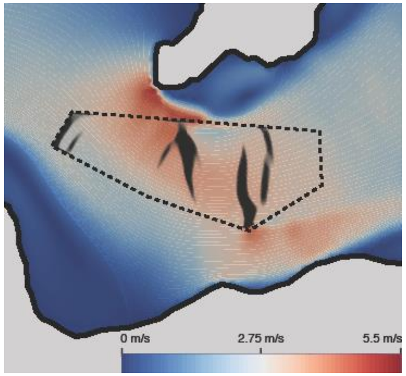
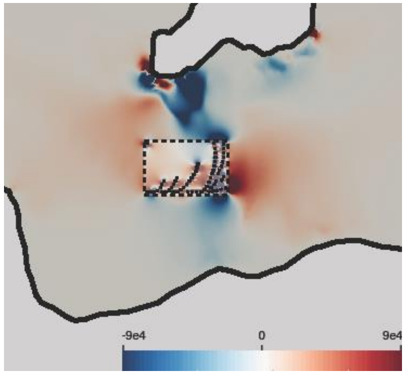
Conclusions

- **Array size and micro-siting design affects power extracted from site**
  - Need to be accounted for in resource assessment methodology
  - Requires an optimisation approach
- **Optimisation is computationally intensive**
  - Use of gradient methods
  - Surrogate-based optimisation
- **Sensitivity analysis highlights importance of accounting for uncertainty**
  - Important to give ‘error bars’ on estimated resource
  - Even better – optimise to balance uncertainty against yield





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DM Culley, SW Funke, SC Kramer, MD Piggott (2015). Tidal stream resource assessment through optimisation of array design with quantification of uncertainty, *accepted to The Proceedings of the 11th European Wave and Tidal Energy Conference*, Nantes, 2015.

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